

# Credit Constraints, Trade and Wealth Distribution\*

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## Abstract

In this paper I study the effects of credit constraints on aggregate outcomes and wealth distribution in closed and open economies. Entrepreneurs, who are heterogeneous in ability and wealth, might encounter collateral constraints as an obstacle to building businesses of optimal size. In the open economy entrepreneurs can decide to enter a foreign market, possibly subject to paying a fixed cost. The main finding of the paper is that while credit frictions negatively affect both closed and open economies, the impact is relatively larger in trading economies when entering the foreign market is costly. Stated differently, credit frictions lower the welfare gains from trade if some entrepreneurs are not exporting.

## 1 Introduction

This paper is motivated by two observations. The first one is a well-documented fact that many firms, especially young ones and especially in developing countries are constrained in their access to credit.<sup>1</sup> Those credit frictions lead to misallocation of resources in production in the economy, lowering aggregate productivity and consequently welfare. Recent quantitative literature argues that the size of those losses can indeed be large.<sup>2</sup>

The second observation is that increased exposure to international trade has uneven effect on domestic firms. The insight of the Melitz (2003) model, strongly supported by the data, is that with trade liberalization the most productive firms to begin with expand further relative to their less successful competitors.

The purpose of this paper is to study the interactions between credit constraints, wealth distribution and openness to international trade. The mechanism that links the three can be best described at the level of individual entrepreneurs. When credit frictions are binding, firms are unable to operate at their optimal scale which affects their profits. Since firms are owned by entrepreneurs,

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<sup>1</sup>See e.g. Banerjee and Duflo (2005).

<sup>2</sup>See Hsieh and Klenow (2009) and Buera et al. (2009).

credit frictions affect the distribution of income and wealth among business owners. But to the extent that an entrepreneur's individual wealth is often used as a collateral, wealth distribution itself affects the prevalence of credit constraints. Openness to international trade by differentially affecting the demand for credit for different entrepreneurs could potentially exacerbate or alleviate the negative impact of credit frictions relative to the case of the closed economy.

To investigate these interactions I first build a stylized model of entrepreneurship. Each agent in the economy is a worker-entrepreneur that runs a company which has an optimal scale depending on the agent's ability. Due to the collateral constraints, entrepreneurs with good ideas but little assets are forced to operate at a suboptimal level. The model features dynamics in that an entrepreneur leaves a bequest for his offspring, who becomes an entrepreneur next period. Inherited wealth in conjunction with a parameter describing the degree of credit imperfections determine the maximum amount an entrepreneur can borrow. The analysis focuses on a stationary equilibrium of this economy, the existence and uniqueness of which is established.

Improving the functioning of credit markets predictably has beneficial impact on the aggregate quantities in the economy. In the baseline version of the model a stronger result is established: with better credit markets the stationary distribution of wealth first order stochastically dominates distributions with higher credit frictions. Thus regardless of the impact of improved access to credit on inequality (which in fact can be ambiguous) decreasing credit frictions is always socially beneficial for all reasonable social welfare functions.

Next the model is extended to the case of trade between two symmetric countries. Comparing the open and closed economy for a fixed degree of credit frictions, international trade not only increases aggregate output and wealth but unambiguously increases welfare. Even though this is a second-best world with multiple distortions, and despite higher inequality in the open economy, opening to trade is necessarily beneficial. The reason is that the wealth distribution in the open economy dominates the distribution in the closed economy in the first order stochastic sense.

The most interesting results of the paper are perhaps those relating to the *differential* impact of credit constraints in the closed and open economies. In both closed and open economies credit frictions have a detrimental impact on the aggregates. However, in the presence of fixed costs of exporting the impact of credit frictions on the open economy is relatively larger than the impact on the closed economy. Another way of phrasing this result is that gains from trade are decreasing in the degree of credit frictions. While I am unable to establish this result analytically, it always holds across a wide range of simulations. The importance of selection into exporting is crucial for this result: without fixed costs of foreign activity credit constraints are shown to be just as important in the open economy as in the closed economy.

The results summarized above are derived in a model that for tractability makes a few strong assumptions. Most notably, it features a "warm-glow" bequest motive and a distribution of ability across members of the same lineage that is uncorrelated over time. These are potentially problematic, since the recent literature studying the aggregate effects of credit frictions (Moll (2009);

Midrigin and Xu (2010)) emphasizes that persistence in productivity is a key element in quantifying losses from frictions. The reason is that with sufficiently persistent productivity shocks, forward-looking agents might overcome their initial constraints with self-financing by saving. In Section 4 I therefore numerically examine the robustness of my results to allowing for correlated shocks and forward-planning agents. A simple calibration of the extended model with infinitely lived agents and persistent productivity shocks suggests that the main insights from the stylized model survive in the richer framework.

## Related Literature

This paper is related to a few diverse strands of the literature. The dynamics of the baseline model draws on macro and development studies that investigate the endogenous determination of wealth distribution in the presence of credit constraints in a closed economy, such as Banerjee and Newman (1993); Aghion and Bolton (1997); Piketty (1997); Lloyd-Ellis and Bernhardt (2000). Banerjee and Duflo (2005) survey this class of models.

This work also shares interests with the more recent studies of aggregate TFP losses from credit constraints. In addition to the work already cited, a series of papers by Buera and Shin as well as Townsend and coauthors fall into this category.

All the studies mentioned so far consider closed economies producing a single good.<sup>3</sup> My paper makes a contribution by analyzing how credit constraints effect the economy in a setting with international trade and monopolistic competition.

In the trade literature there is a growing number of papers analyzing the implications of credit frictions with heterogeneous firms. The static version of my model bears a resemblance to the work of Chaney (2005). In his model firms receive liquidity shocks and as a consequence some potentially profitable exporters are prevented from entering the foreign market. Conditional on exporting, the intensive margin is not distorted by frictions, however. My model considers the other extreme, in which an entrepreneur can borrow a certain amount, regardless if it is used to cover fixed or operating costs. Manova (2010) also investigates a case in which firms are constrained in financing both fixed and variable costs. The models of Chaney and Manova are essentially static, however. This is not a great problem in their respective applications: Chaney considers the effects of exchange rate movements on the set of exporters and Manova is most interested in testing the predictions of her model for the cross-section of countries and industries. In this work I am interested in how aggregate quantities in the economy react to credit constraints for different counterfactual scenarios (autarky, trading economy). I believe that handling this problem convincingly necessarily requires a dynamic model. To see why, consider the related model with entrepreneurs and borrowing constraints of Foellmi and Oechslin (2010). They take as a starting point a fixed wealth distribution and look at the resulting income distribution under two scenarios: closed economy and small open economy. One

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<sup>3</sup>A notable exception is Buera et al. (2009) who analyze a two-sector economy. That paper is particularly close to my work in that it also emphasizes the role of fixed costs.

problem is that with their particular way of modeling trade integration, market structure changes completely so it is not clear what the comparison entails. Moreover, the initial wealth differences are the only source of heterogeneity in the economy. If their economy was to continue for more than one period the initial differences would disappear over time under many standard approaches to modeling intertemporal dynamics. A static model can be useful for evaluating short-run responses to trade liberalization, but to assess long-run consequences we need a dynamic model.

Dynamic trade models with credit constraints are featured for example by Ranjan (2001) and Antràs and Caballero (2009). Those, however, rely on variants of Heckscher–Ohlin forces and as such have different interests as my paper. This model shares the modeling of the entire population of the country as workers-entrepreneurs with Itskhoki (2008), but again the focus of the papers is very different.

## 2 Closed Economy

In this section I characterize the stationary equilibrium in the closed economy. After outlining the demographics of the model, I describe the solution to the static problem faced by entrepreneurs with perfect credit markets and with collateral constraints. The dynamics works through wealth accumulation and individual productivity shocks. However, the structure of the problem faced by entrepreneur each period is the same.

### 2.1 Demographics

The demographical structure is given by successive generations. Each period a mass  $L$  of agents is born and each agent lives for one period only. Upon birth, an individual receives bequest  $w$  in the form of physical capital left by his parent. Each agent becomes a worker-entrepreneur. Entrepreneurs are heterogeneous in their ability  $\theta$ .<sup>4</sup> After observing  $(w, \theta)$  the agent decides how much to invest in his company, possibly borrowing some funds or lending some of his inherited wealth. At the end of the period, after production has been completed and profits and interest income realized, the agent decides how to split his end-of-period wealth between consumption and bequest for his single descendant. I assume a simple specification of the utility function:

$$U = c^{1-\alpha} b^\alpha,$$

where  $c$  is consumption and  $b$  is a bequest left to the offspring, both in terms of one final consumption/capital good. This utility function represents the “warm glow” bequest motive and is adopted here for tractability reasons.<sup>5</sup> The feature of this specification that a constant fraction of wealth

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<sup>4</sup>“Entrepreneurial ability” and “productivity” are used interchangeably throughout the paper.

<sup>5</sup>As in much of the similar literature, see e.g. Banerjee and Newman (1993); Piketty (1997); Ranjan (2001).

is left as bequest, while analytically convenient is somewhat restrictive. Section 4.2 numerically analyzes the case with infinitely-lived agents (or perfectly altruistic dynasties) instead.

## 2.2 Technology

An entrepreneur with ability  $\theta$  can transform  $k_i$  units of capital into  $(\theta k_i)^\gamma$  units of the differentiated intermediate input with no capital depreciation<sup>6</sup>:

$$k_i \text{ capital} \rightarrow \begin{cases} (\theta k_i)^\gamma & \text{intermediate output} \\ k_i & \text{capital} \end{cases} .$$

I assume that  $0 < \gamma < 1$  so that there are decreasing returns to scale in production at the individual level.<sup>7</sup> Horizontal differentiation in the intermediate sector is costless so each agent produces a distinct variety.

The final good can be used for consumption or used as capital. It can be produced using two technologies. First, it can be costlessly assembled from intermediates with a CES aggregator

$$Q = \left[ \int_0^L q_i^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} .$$

Alternatively, the final good can be produced by a “homogeneous” sector that transforms a unit of capital into  $r$  units of final output without capital depreciation. It will be assumed that parameters are such that the homogeneous sector is operating.<sup>8</sup> The purpose of the “homogeneous” sector is to pin down the interest rate. An exogenously fixed interest rate simplifies the analysis considerably. This assumption also will be relaxed in the numerical Section 4.2.

Borrowing and lending occurs through an intermediary that is perfectly competitive in that it offers a single interest rate  $r$ .

Throughout the paper price of the final good is normalized to one.

## 2.3 Static Problem

In this subsection I describe how the equilibrium is determined within the period. But first I briefly discuss the aggregate accounting issues. Let  $Y$  denote the aggregate revenue of producers of intermediates.  $Y$  is thus also the cost and the revenue of the sector costlessly assembling intermediates into the final good. Denoting by  $K^H$  the capital employed in the homogeneous sector, by  $K^D$  capital used in the differentiated sector and by  $K = K^H + K^D$  total capital stock, total income (GDP) in a country is equal to  $Y + rK^H$ .

<sup>6</sup>Introducing capital depreciation requires only a minor modification of notation and is incorporated in the numerical Section 4.2.

<sup>7</sup>This assumption simplifies the determination of aggregates in the dynamic setting. It is standard in entrepreneurial models with capital as the only factor input.

<sup>8</sup>An explicit restriction on the parameters for this to be the case will be provided.

The nominal output of the differentiated sector equals  $Y = \int_0^L p(q_i) q_i di$ . With CES demand we can express the demand for goods provided by an entrepreneur  $i$  as:

$$q_i = \frac{Y}{P} \left( \frac{p_i}{P} \right)^{-\sigma},$$

where  $P = \left[ \int_0^L p_i^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$  is the minimum cost of assembling a unit of final good using intermediate inputs. But with price of the final good normalized to one, we have  $P \equiv 1$  so  $Y$  is also the real output of the differentiated sector:

$$Y = \left[ \int_0^L [(\theta k_i)^\gamma]^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}. \quad (1)$$

Without frictions in the credit market, optimal choice of each entrepreneur depends only on his entrepreneurial ability but not on his inherited wealth. Standard calculation give the expressions for optimal investment size:

$$\tilde{k}(\theta) = \left( \frac{1}{\gamma} \frac{\sigma}{\sigma-1} r \right)^{\frac{-\sigma}{\phi}} Y^{\frac{1}{\phi}} \theta^{\frac{\gamma(\sigma-1)}{\phi}} \quad (2)$$

and profits:

$$\tilde{\pi}(\theta) = Y^{\frac{1}{\phi}} \theta^{\frac{\gamma(\sigma-1)}{\phi}} \left( \frac{1}{\gamma} \frac{\sigma}{\sigma-1} r \right)^{\frac{-\gamma(\sigma-1)}{\phi}} \frac{\phi}{\sigma} \quad (3)$$

of an agent with ability  $\theta$ , where  $\phi \equiv \sigma - \gamma(\sigma - 1) > 0$  is a new variable defined to simplify notation.

Combining (1) and (2) we can find an explicit expression for the output of the differentiated sector:

$$Y = \left( \frac{1}{\gamma} \frac{\sigma}{\sigma-1} r \right)^{\frac{-\sigma\gamma}{\sigma(1-\gamma)}} \left\{ L^{\frac{\sigma}{\sigma-1}} \left[ \mathbf{E} \left( \theta^{\frac{\gamma(\sigma-1)}{\phi}} \right) \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\frac{\phi}{\sigma(1-\gamma)}}. \quad (4)$$

This solution is valid as long as there is net supply of capital to the homogeneous sector. The restriction on parameters that guarantees that in the stationary equilibrium we indeed have  $K \geq K^D$  is derived in Appendix A.1.1 and is maintained as an assumption throughout this section.

## Credit Constraints

The main interest of this paper lies in situations where entrepreneurs are unable to obtain sufficient funds to operate at their optimal scale. Specifically, I assume a simple form of collateral constraints, in which the maximum investment an agent can undertake is a multiple of his inherited wealth:

$$k \leq \lambda w, \quad \lambda \geq 1. \quad (5)$$

This way of modeling credit frictions captures the notion that the ability to borrow depends on entrepreneur's own funds while being very tractable, hence its widespread use in the literature.<sup>9</sup> It can be provided with the following micro foundation: to borrow funds from the intermediary an entrepreneur needs to provide his wealth as (interest-bearing) collateral. Once he obtains capital  $k$  from the lender, he can run away with a fraction  $\frac{1}{\lambda}$  of it. The only punishment is the loss of his collateral. In that case the maximum amount lenders would be willing to advance would be bound by  $w \geq \frac{1}{\lambda}k$ , which is just (5). The two extreme cases are  $\lambda = \infty$  corresponding to the absence of credit frictions and  $\lambda = 1$  in which case there is no credit at all.

Let  $\tilde{w}(\theta)$  be the minimum amount of wealth an agent needs to undertake investment of optimal size for his ability. We simply have

$$\tilde{w}(\theta) = \frac{1}{\lambda} \tilde{k}(\theta).$$

If an entrepreneur's wealth is less than  $\tilde{w}(\theta)$  he will be credit constrained in that he would like to borrow more but is unable to secure additional credit. Since for a constrained agent the marginal revenue exceeds the marginal cost, he will necessarily invest up to his limit. Due to the credit frictions, the amount an agent with ability  $\theta$  and inherited wealth  $w$  actually invests is:

$$k(w, \theta) = \begin{cases} \lambda w & \text{if } w < \tilde{w}(\theta) \\ \tilde{k}(\theta) & \text{if } w \geq \tilde{w}(\theta) \end{cases}.$$

Therefore we can write profits from operating the firm as follows:

$$\pi(w, \theta) = \begin{cases} (\theta \lambda w)^{\frac{\gamma(\sigma-1)}{\sigma}} Y^{\frac{1}{\sigma}} - \lambda w r & \text{if } w < \tilde{w}(\theta) \\ \tilde{\pi}(\theta) & \text{if } w \geq \tilde{w}(\theta) \end{cases}.$$

The total income of an agent comprises in addition his interest income:

$$y(w, \theta) = \begin{cases} (\theta \lambda w)^{\frac{\gamma(\sigma-1)}{\sigma}} Y^{\frac{1}{\sigma}} - (\lambda - 1) w r & \text{if } w < \tilde{w}(\theta) \\ \tilde{\pi}(\theta) + w r & \text{if } w \geq \tilde{w}(\theta) \end{cases}.$$

Figure 1 plots income as a function of inherited wealth for two values of productivity  $\theta$ . The function is concave for  $w < \tilde{w}(\theta)$  reflecting the decreasing returns in production and CES demand, and is linear above  $\tilde{w}(\theta)$  as in that range any extra wealth is simply rented out at interest rate  $r$ .

Aggregate demand  $Y$  in the expressions presented above is an endogenous object. Unlike the case without credit frictions, it cannot in general be computed without knowing the entire distribution of wealth among agents. Now we turn to the question how this distribution is determined.

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<sup>9</sup>E.g. Evans and Jovanovic (1989); Buera and Shin (2008); Moll (2009)

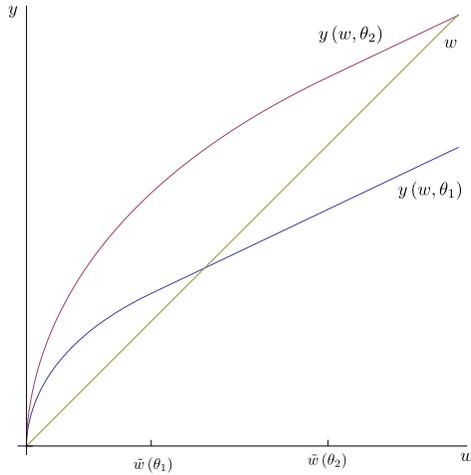


Figure 1: Income as a Function of Inherited Wealth,  $\theta_2 > \theta_1$

## 2.4 Stationary Equilibrium

The evolution of wealth within a given lineage follows a clear pattern. An agent born at time  $t$  with inherited wealth  $w_t$  and ability  $\theta_t$  receives income (from running his company and interest on initial wealth)  $y_t(w_t, \theta_t)$  and leaves a constant fraction  $\alpha$  (a result of the assumed preferences) of his end of period wealth as a bequest, which becomes the initial wealth for his descendant next period:  $b_t(w_t, \theta_t) = \alpha [y_t(w_t, \theta_t) + w_t] = w_{t+1}$ . In general the evolution of this economy is nevertheless complicated: how much an individual entrepreneur leaves as a bequest depends on the entire wealth distribution (hence the  $t$ -subscripts on the  $y$  and  $b$  functions). The reason is that the wealth distribution determines who is credit constrained, affecting aggregate output  $Y$  which in turn influences profits of individual entrepreneurs. Therefore I concentrate on the stationary equilibrium of this economy - an equilibrium in which all aggregate quantities are time invariant and the distribution of wealth across agents itself is time invariant as well.

Before proceeding further I need to discuss the adopted process for entrepreneurial ability. I am assuming that  $\theta$  is drawn i.i.d. across agents within a given cohort and across members of the same lineage from a fixed distribution with support  $[\underline{\theta}, \bar{\theta}]$ .<sup>10</sup> That ability is uncorrelated among members of the same lineage is a particularly strong assumption. However, as will be discussed more thoroughly in Section 4, it can be relaxed without changing the qualitative results presented below.

The existence of a stationary equilibrium and associated invariant distribution is still by no means obvious. The lengthy proof of the following proposition is contained in the Appendix.

**Proposition 1.** *For any  $\lambda \geq 1$  there exists a unique stationary equilibrium of the economy.*

There are two main steps in the proof. The first one is to establish that fixing the aggregate

<sup>10</sup>See Assumption 2 in the Appendix for precise conditions.

demand (which agents takes as given) at any level  $Y_0$  the wealth distribution converges to a unique invariant distribution. The output of the differentiated sector evaluated using this distribution and demand  $Y_0$  will in general be different from  $Y_0$  and take some value  $Y_1 = \zeta(Y_0; \lambda)$ . The second part of the proof establishes that there exists a fixed point  $Y^*(\lambda) = \zeta(Y^*(\lambda); \lambda)$  and that it is unique.

Since for any degree of credit imperfections the stationary equilibrium is unique, we can meaningfully compare equilibrium outcomes for different values of  $\lambda$ . The following proposition presents the main comparative statics result of this section.

**Proposition 2.** *If  $\lambda_2 > \lambda_1$  then the wealth distribution in the stationary equilibrium of the economy with  $\lambda = \lambda_2$  first order stochastically dominates the distribution in the stationary equilibrium of the economy with  $\lambda = \lambda_1$ . Aggregate output of the differentiated sector and aggregate capital are (weakly) higher in the economy with better credit markets:  $Y^*(\lambda_2) \geq Y^*(\lambda_1)$ ,  $K(\lambda_2) \geq K(\lambda_1)$ .*

The fact that improving the functioning of the credit markets has a positive impact on aggregates (output of the differentiated sector, GDP and aggregate wealth) is not surprising. There are welfare gains in a stronger sense, however. Since in the model consumption is directly related to wealth, first order stochastic dominance implies that everybody is expected to be better off when credit markets work better.<sup>11</sup> Thus any sensible social welfare function would rank higher the equilibrium with less credit frictions.<sup>12</sup>

In particular, this is true despite the fact that better credit markets might result in higher inequality. In fact numerical simulations suggest that wealth inequality typically goes up as we increase  $\lambda$ . However, it is possible to construct cases in which inequality goes down with better credit.<sup>13</sup> The ambiguity stems from the fact that better access to credit can have conflicting effects. Fixing the wealth distribution, higher  $\lambda$  would tend to increase “between-group” income inequality, that is inequality between agents with different abilities. The reason is that the most productive agents want to make the biggest investments, so they are most likely to be constrained. Thus those who would be successful anyway stand to gain the most from better credit. On the other hand, better credit tends to lower “within-group” inequality, because with better credit firm profits among entrepreneurs with the same ability are more similar. The fact that the wealth distribution changes makes the forces more complicated (in fact both between- and within-group inequality can go up with higher  $\lambda$ ). Nevertheless, while the effect of  $\lambda$  on inequality is ambiguous, its effect on welfare

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<sup>11</sup>Individual utility is linear in end-of-period wealth. Distribution of wealth at the beginning of period is just a version of the end-of-period distribution scaled by  $\frac{1}{\alpha}$  in a stationary setting. It is thus appropriate to look at the invariant wealth distribution in drawing welfare conclusions.

<sup>12</sup>The usual disclaimer about comparing welfare among different equilibria applies in principle. However, in all numerical simulations of a transition path from a stationary equilibrium with  $\lambda_1$  to an equilibrium with  $\lambda_2 > \lambda_1$ , the wealth distribution is monotonically increasing (in the f.s.d. sense) on the way to the new invariant distribution. Thus if we were to calculate welfare gains from increasing  $\lambda$  taking into account the transitional dynamics to the new equilibrium the conclusion would not be altered.

<sup>13</sup>This can happen when the underlying ability distribution has a hump-shaped density (e.g. a truncated lognormal) or increasing density (which is clearly empirically unappealing). For distributions with decreasing density (e.g. truncated Pareto) the inequality appears to be increasing in  $\lambda$ .

is not.

### 3 Open Economy

In this section I investigate the properties of the model in the open economy. I start by describing the static problem faced by an entrepreneur in a world with two symmetric countries. Next I show that the existence, uniqueness and equilibrium comparative statics results from the closed economy have their open economy analogs. Finally I move to comparing equilibrium outcomes in the open and closed economies which are the central results of this paper.

#### 3.1 Static Problem

Consider a world consisting of two countries, indexed  $H$  and  $F$  for Home and Foreign that are in all respects symmetric. As before, there are no barriers to entering the entrepreneur's home market, therefore all agents will be active sellers in their country. International trade, in contrast, is costly. All shipments are subject to the standard iceberg costs  $\tau$ . Moreover, entering the foreign market requires a fixed cost of  $f_x Y^{\frac{1}{\phi}}$  units of capital. Notice that the fixed cost is assumed to depend on the foreign market size, which can be justified as reflecting e.g. marketing cost as in Arkolakis (2009). The particular functional form assumption implies that the elasticity of the fixed cost with respect to the market size is the same as the elasticity of operating profits of unconstrained agents. Consequently, the net profits of unconstrained agents rise at a constant rate with respect to the market size. This feature simplifies the analysis without affecting the qualitative results.

An entrepreneur that has decided to make a total investment of size  $k$  can do so in two ways. He can decide to sell on the domestic market only, in which case the entire capital is put into production, generating output  $(\theta k)^\gamma$  and the entrepreneur keeps  $k$  units of capital as there is no depreciation. Alternatively, he might decide to export. Only the part of investment left after paying the fixed cost is put into physical production, generating  $\left[\theta \left(k - f_x Y^{\frac{1}{\phi}}\right)\right]^\gamma$  units of output of the entrepreneur's variety of intermediates. I assume the fixed cost can be liquidated at no loss at the end of the period, so the agent still keeps  $k$  units of capital.

Now consider the problem of how an exporting entrepreneur splits his output between home and foreign market once it has been produced. Optimal choice will equalize marginal revenue with respect to quantity shipped to each country. Suppose the overall output of good is  $X$ . Then the producer should sell  $x_H$  at home and  $x_F$  in foreign (which means shipping  $\tau x_F$ ) so that:

$$\frac{x_H}{x_F} = \tau^\sigma \frac{Y_H}{Y_F} \left(\frac{P_H}{P_F}\right)^{\sigma-1}.$$

In particular, in the symmetric case with  $Y_H = Y_F = Y$ ,  $P_H = P_F \equiv 1$  we have  $x_H = \tau^\sigma x_F$ . Total revenue from producing  $X$  units of output is then  $X^{\frac{\sigma-1}{\sigma}} Y^{\frac{1}{\sigma}} (1 + \tau^{1-\sigma})^{\frac{1}{\sigma}}$ .

We still need to determine which entrepreneurs will decide to export. Because of decreasing returns at the firm level, this is not exactly a standard Melitz-type problem where the cutoff exporter is determined from making zero export profits. The reason is that production cannot be split between production for home and foreign markets - there is joint production and only output can be split.

Consider first an economy without credit frictions. If an entrepreneur produces only for the domestic market he makes profits  $\tilde{\pi}_H(\theta)$  given in (3). If he decides to export, we can represent his problem as optimally choosing total physical output  $X = x_H + \tau x_F = (\theta k_v)^\gamma$  to maximize profits, or equivalently:

$$\max_{k_v} (\theta k_v)^\gamma \frac{\sigma-1}{\sigma} Y^{\frac{1}{\sigma}} (1 + \tau^{1-\sigma})^{\frac{1}{\sigma}} - r k_v - r f_x Y^{\frac{1}{\phi}},$$

where  $k_v$  represents the variable input of capital. The optimal variable capital conditional on exporting is :

$$\tilde{k}_{ve}(\theta) = \theta^{\frac{\gamma(\sigma-1)}{\phi}} Y^{\frac{1}{\phi}} \left( \frac{1}{\gamma} \frac{\sigma}{\sigma-1} r \right)^{\frac{-\sigma}{\phi}} (1 + \tau^{1-\sigma})^{\frac{1}{\phi}}$$

and the total capital used by an unconstrained exporter is  $\tilde{k}_E(\theta) = \tilde{k}_{ve}(\theta) + f_x Y^{\frac{1}{\phi}}$ .

An entrepreneur will engage in exporting if *total* profits from exporting will be higher than profits from serving the domestic market only. Using the fact that

$$\tilde{k}_{ve}(\theta) = (1 + \tau^{1-\sigma})^{\frac{1}{\phi}} \tilde{k}_H(\theta),$$

where  $\tilde{k}_H(\theta)$  is the optimal investment for domestic market given in (2), we can express operating profits of an exporter as

$$\tilde{\pi}_E^o(\theta) = (1 + \tau^{1-\sigma})^{\frac{1}{\phi}} \tilde{\pi}_H(\theta). \quad (6)$$

We find the cutoff ability  $\theta_{ue}$  for an unconstrained exporter by equating total profits from exporting and domestic-only sales:

$$\tilde{\pi}_E^o(\theta_{ue}) - r f_x Y^{\frac{1}{\phi}} = \tilde{\pi}_H(\theta_{ue}).$$

Making use of (6) and (3) this equation can be explicitly solved for the cutoff  $\theta_{ue}$ :

$$\theta_{ue} = \left( \frac{\sigma}{\phi} r f_x \right)^{\frac{\phi}{\gamma(\sigma-1)}} \left( \frac{1}{\gamma} \frac{\sigma}{\sigma-1} r \right) \left\{ \left[ (1 + \tau^{1-\sigma})^{\frac{1}{\phi}} - 1 \right] \right\}^{-\frac{\phi}{\gamma(\sigma-1)}}.$$

If an access to credit was unrestricted, all entrepreneurs with ability above  $\theta_{ue}$  would become exporters, while those with lower ability would sell only in their home market. Observe that the cutoff does not depend on the size of the differentiated sector  $Y$ . This property, which simplifies

the analysis in the dynamic setting, follows from the assumed specification of fixed costs depending on the market size.

To summarize, the overall optimal level of investment in the open economy without credit frictions is:

$$\tilde{k}(\theta) = \begin{cases} (1 + \tau^{1-\sigma})^{\frac{1}{\phi}} \tilde{k}_H(\theta) + f_x Y^{\frac{1}{\phi}} & \text{if } \theta \geq \theta_{ue} \\ \tilde{k}_H(\theta) & \text{if } \theta < \theta_{ue} \end{cases}.$$

There is a discrete jump in investment, revenue and operating profits at  $\theta = \theta_{ue}$  but net profits are a continuous function of  $\theta$ .

Similarly as in the closed economy, when the supply of capital is sufficiently high the aggregate output of the frictionless economy can be computed without knowledge of the wealth distribution. The formula for  $Y$  in this case and the required restriction on parameters is presented in Appendix A.2.1.

## Credit Constraints

Credit frictions take the same form as in the closed economy: an entrepreneur's total investment size is constrained to be at most  $\lambda$  times his initial wealth. As far as credit is concerned, there is no distinction between financing the fixed cost of exporting and operating costs. The level of wealth required to finance optimal investment again is  $\tilde{w}(\theta) = \frac{\tilde{k}(\theta)}{\lambda}$  and it exhibits a discrete jump at the exporting cutoff  $\theta_{ue}$ . I now describe the behavior of credit constrained entrepreneurs, or those with wealth below  $\tilde{w}(\theta)$ . Agents that could not make profits from exporting even in the unrestricted case clearly do not export when they are constrained. So for those with ability below  $\theta_{ue}$  we have the same type of solution as in the closed economy: they invest up to the borrowing limit and produce for domestic market only.

The situation is more complicated for agents with  $\theta > \theta_{ue}$ . They might or might not find it profitable to enter exporting. For each  $\theta > \theta_{ue}$  there is a threshold wealth  $w_e(\theta)$  such that agents with wealth below  $w_e(\theta)$  will not export while those with  $w_e(\theta) \leq w < \tilde{w}(\theta)$  will enter the foreign market. Derivations summarized in Section A.2.2 of the Appendix show that the threshold wealth required for exporting takes the form:

$$w_e(\theta) = \begin{cases} w_e^1(\theta) & \text{for } \theta_{ue} \leq \theta < \theta_{pde} \\ w_e^0 & \text{for } \theta \geq \theta_{pde} \end{cases}, \quad (7)$$

where  $w_e^0$ ,  $w_e^1(\theta)$  and  $\theta_{pde}$  are defined in the Appendix. Figure 2 presents the partition of entrepreneurs in the  $\theta - w$  space. There is a number of qualitatively different regions in that graph. The least productive agents ( $\theta < \theta_{ue}$ ) never export and are constrained if their wealth is below  $\tilde{w}(\theta) = \frac{\tilde{k}_H(\theta)}{\lambda}$ . Among the most productive group ( $\theta \geq \theta_{pde}$ ) three types of behavior are possible.

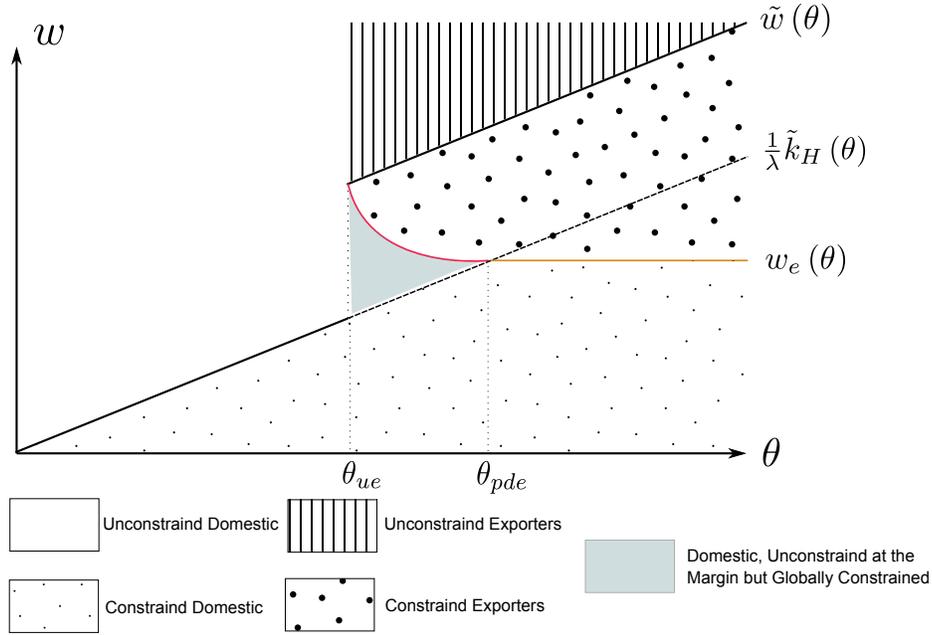


Figure 2: Entrepreneurs in  $w - \theta$  Space

Those who inherited little ( $w < w_e^0$ ) find that their scarce wealth is too precious to be spent on the fixed costs of exporting. With sufficient wealth ( $w_e^0 \leq w < \tilde{w}(\theta)$ ) they become exporters but they operate at an inefficient scale. The wealthiest can run their companies at an unconstrained exporter level. For entrepreneurs with intermediate abilities ( $\theta_{ue} \leq \theta < \theta_{pde}$ ) there are yet more possibilities. The extremes are clear: the poorest serve the domestic market at the constrained level and the richest export and are unconstrained. Entrepreneurs with wealth satisfying  $w_e^1(\theta) \leq w < \tilde{w}(\theta)$  decide to enter the export market but lack funds to produce as much as they would want to. The most interesting case is of those agents who inherited wealth in the interval  $\frac{1}{\lambda} \tilde{k}_H(\theta) \leq w < w_e^1(\theta)$ . They sell for the domestic market only and they do it at their first-best home sales level. They are “unconstrained at the margin” - if they were allowed to borrow a little more they would see no benefit from doing so. Yet they are globally constrained - if they were allowed to borrow larger amounts they would happily do so as it would make the entry to the foreign market profitable.<sup>14</sup>

Observe that credit constraints can be viewed as another mechanism for explaining the coexistence of small exporters and big firms that do not export. With high  $\theta$  but low  $w$  an entrepreneur can choose not to export and yet have domestic sales larger than unconstrained exporter with  $\theta$  just above  $\theta_{ue}$ . With two dimensions of heterogeneity, it is not surprising the link of the Melitz model between firm’s performance on the domestic market and exporting decision can be broken.

<sup>14</sup>Those firms are similar to the liquidity-constraints firms of Chaney (2005).

### 3.2 Stationary Equilibrium

Extending Propositions 1 and 2 to the case of an open economy requires only minor modifications of their proofs, as described in more detail in Appendix A.2. Thus despite the indivisible nature of the fixed exporting cost, there still exists a unique stationary equilibrium and the results on comparative statics with respect to the degree of credit imperfections  $\lambda$  are unchanged.

The interest of this paper lies in comparing the equilibrium outcomes in the closed and open economies. The following proposition establishes that there are gains from international trade.

**Proposition 3.** *For any fixed  $\lambda \geq 1$  the wealth distribution in the stationary equilibrium of the open economy first order stochastically dominates the distribution in the stationary equilibrium of the closed economy. Aggregate output of the differentiated sector and aggregate capital are higher in the open economy:  $Y_o^*(\lambda) > Y_c^*(\lambda)$ ,  $K_o(\lambda) > K_c(\lambda)$ .*

First order stochastic dominance of the wealth distribution represents a very strong form of the gains from trade. Not only do aggregate output and capital grow, but an individual of any ability can expect to inherit a higher bequest, have higher income and hence consume more in the open economy than under autarky. Proposition 3 is true as long as there are some exporters in the economy - that is what is understood by an open economy in a symmetric setting. As will become clear shortly, it is important for matters of interest to this paper whether all entrepreneurs export or only a subset of them. The following proposition is important for discussing the consequences of selection into exporting.

**Proposition 4.** *Suppose there are no fixed costs of exporting ( $f_x = 0$ ). Then for any  $\lambda \geq 1$  the wealth distribution in the open economy is just a scaled-up version of the distribution in the closed economy, where the scaling does not depend on  $\lambda$ . More precisely,  $G_c^*(w) = G_o^*\left(\left(1 + \tau^{1-\sigma}\right)^{\frac{1}{(\sigma-1)(1-\gamma)}} w\right)$ , where  $G_i^*$  denotes the CDF of the invariant distribution of wealth in the stationary equilibrium.*

One immediate corollary of Proposition 4 is that when there are no fixed costs of exporting so that all entrepreneurs enter the foreign market, all (scale-independent) measures of inequality are the same in the open as in the closed economy. In this case opening to trade does not affect wealth (and income) inequality at all.

There is an overwhelming evidence that the empirically relevant case features selection into exporting based on measures of productivity. What are the implications for inequality of opening to trade in the presence of extensive margin (positive fixed costs in the model)? I conjecture that with an active extensive margin, inequality is higher in the open economy than in the closed economy for any degree of credit frictions.<sup>1516</sup> Unfortunately, I cannot provide a formal proof turning that

<sup>15</sup>I use “positive fixed costs”, “active extensive margin” and “selection into exporting” interchangeably as is common in the literature based on the Melitz model. Strictly speaking what I mean is  $f_x > 0$ . Even when fixed costs are positive but so small that  $\theta_{ue} < \underline{\theta}$  (everybody exports when unconstrained) the results of this paper hold.

<sup>16</sup>An analogous result is shown in a static model with labor market frictions by Helpman et al. (2010).

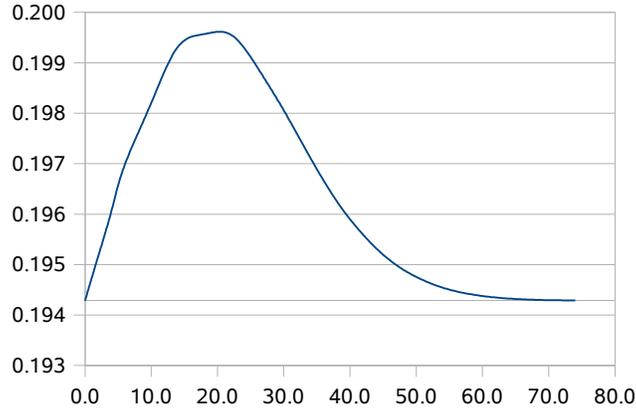


Figure 3: Gini Coefficient of Wealth as a Function of Fixed Costs

conjecture into a proposition. But the conjecture was confirmed in all numerical simulations for a wide range of parameters. Figure 3 presents as an example the Gini coefficient for wealth as a function of  $f_x$  for one such simulation. Why does inequality rise in open economy when  $f_x > 0$ ? It can be shown that when there are no fixed costs of entering the foreign market, opening to trade has similar consequences for entrepreneur's income as a particular change in his wealth.<sup>17</sup> Importantly, the form of this relationship is the same for all agents. That is why the distribution of wealth in the open economy is just a shifted distribution from the closed economy.<sup>18</sup> When there are positive fixed export cost, openness has more asymmetric effects on different agents. Since the size of the fixed cost is independent of firm's characteristics, conditioning on inherited wealth fixed costs are less of a burden for more productive agents. The already most lucky agents can get the extra income from exporting while the less able entrepreneurs are stuck with serving the domestic market only. Even among exporters net income rises relatively more for the more productive entrepreneurs. This is why inequality goes up in the open economy. Notice that this logic does not depend on the degree of credit frictions.

Similarly as was the case with improving credit markets, increased inequality resulting from openness is not a reason to oppose trade liberalization on social welfare grounds. Proposition 3 ensures that welfare is higher in the open economy for any standard social welfare function.

<sup>17</sup>This fact is used in the proof of Proposition 4.

<sup>18</sup>If we consider the transitional dynamics between the stationary equilibria, inequality would be changing over time. But it is the same in the starting and in the final point.

### 3.3 Impact of Credit Constraints and the Degree of Openness

In the previous sections we have established that in both closed and open economies credit constraints negatively affect aggregate outcomes and individual welfare.<sup>19</sup> In this part I consider the main question of interest of this paper: is there a *differential* impact of credit constraints on the open economy relative to autarky?

One way to evaluate this differential impact is to consider the following experiment: suppose we can improve the functioning of the credit markets so that  $\lambda$  increases from some initial value  $\lambda_1$  to  $\lambda_2 > \lambda_1$ . We know already that in both open and closed economy the output in the new stationary equilibrium would be higher after the reform (as long as the constraint was initially binding for some agents). But will it increase relatively more in the open economy, so is it the case that:

$$\frac{Y_c^*(\lambda_2)}{Y_c^*(\lambda_1)} < \frac{Y_o^*(\lambda_2)}{Y_o^*(\lambda_1)}. \quad (8)$$

The question of the sensitivity of aggregates to credit imperfections in open and closed economies might be of main interest to macroeconomists. But observe that the same problem might be phrased by a trade economist as asking if the degree of credit frictions affects the gains from trade the economy reaps. Higher sensitivity of output in the open economy is equivalent to gains from trade rising with the development of the credit markets, as a trivial rearrangement of (8) gives:

$$\frac{Y_o^*(\lambda_1)}{Y_c^*(\lambda_1)} < \frac{Y_o^*(\lambda_2)}{Y_c^*(\lambda_2)}. \quad (9)$$

It turns out that the answer to both questions stated above crucially depends on the size of the fixed costs. When  $f_x = 0$  and everybody exports, openness does not lead to increased sensitivity of the economy to credit constraints, as the following corollary of Proposition 4 illustrates.

**Corollary.** *Suppose there are no fixed costs of exporting ( $f_x = 0$ ). Then aggregate losses from credit frictions are the same in the open and closed economy and gains from trade are independent of the degree of frictions:  $\frac{Y_o^*(\lambda)}{Y_c^*(\lambda)} = \frac{K_o(\lambda)}{K_c(\lambda)} = (1 + \tau^{1-\sigma})^{\frac{1}{(\sigma-1)(1-\gamma)}}$ .*

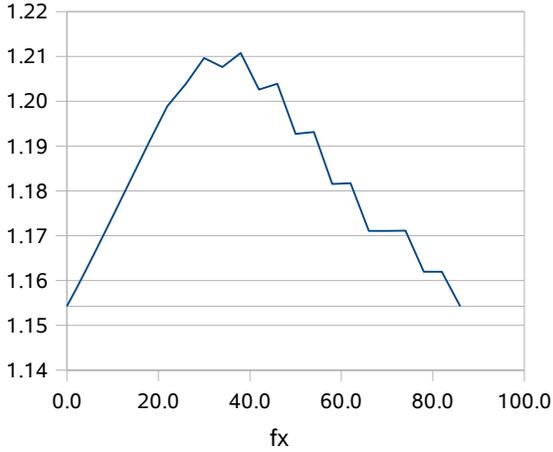
This is no longer true in the presence of positive fixed export costs. I strongly conjecture that the following is true:

**Conjecture.** *With positive fixed costs of exporting efficiency losses from credit frictions are higher in the open economy and gains from trade are increasing in  $\lambda$ .*

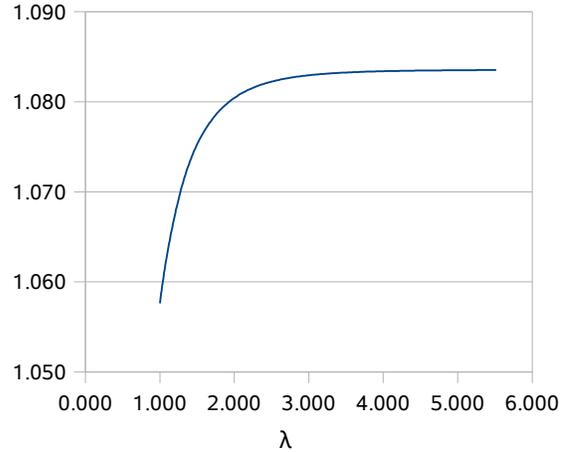
This is perhaps the most interesting claim of the paper. Unfortunately I am unable to provide

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<sup>19</sup>The discussion in this section focuses on the output of the differentiated sector. The same results hold for GDP and aggregate wealth. With individual utility linear in wealth, aggregate capital is a value taken by a utilitarian social welfare function.



(a)  $\frac{Y^*(\lambda_2)}{Y^*(\lambda_1)}$ ,  $\lambda_2 > \lambda_1$  as a Function of Fixed Costs



(b) Gains From Trade in Terms of Output with  $f_x > 0$  as a Function of  $\lambda$

Figure 4: Two Perspectives on Sensitivity of Losses from Credit Frictions to Openness

an analytical proof of that result.<sup>20</sup> Nevertheless, the claim was confirmed in all numerical simulations with different parameter values I have tried. Below I present the results of some simulations and try to offer an intuition why the claim seems to be true.

Figure 4a plots the ratio of outputs of the differentiated sector for two values of  $\lambda$  as  $f_x$  varies from zero to value so high that the economy is in autarky. To understand the shape of that graph it is useful to think about constrained entrepreneurs in a closed economy and in a trading economy without fixed costs. In both cases a marginal increase in  $\lambda$  leads to a proportional increase in investments. As a result revenue of those constrained entrepreneurs rises by the same proportion in both cases. The situation is different when fixed costs of exporting are positive. Again consider the reaction of constrained entrepreneurs to a marginal increase in  $\lambda$ . Total investment increases proportionally as before. But revenues in the open economy rise relatively more than in a closed economy. To see this consider an entrepreneur who already was an exporter. The extra capital he can borrow thanks to higher  $\lambda$  will then go entirely into working capital. For entrepreneurs like him variable capital rises more than proportionally with  $\lambda$  and as a consequence revenues are more sensitive to the degree of credit frictions in the open economy.

Essentially the same intuition explains why gains from trade, measured as the ratio of output of the differentiated sector in the open and closed economies, are increasing in  $\lambda$  when fixed costs are positive. Better credit market increases revenues in both closed and open economy. But the effect is relatively stronger in the open economy because with higher  $\lambda$  a relatively higher fraction of an

<sup>20</sup>Proofs of Propositions 2-4 rely on establishing the dominance of Markov operators  $T^*$  corresponding to different equilibria. There are known methods to make this “levels” comparison. Establishing the differential impact of credit constraints in open and closed economies would require comparing the “slopes” of  $T^*$  operators in autarky and with trade. I am not aware of mathematical tools that can be applied for this purpose in the present context.

entrepreneur’s total investment can be put into productive use and a smaller fraction is devoted to paying off the fixed cost. Figure 4b is a representative plot of  $\frac{Y_o^*(\lambda)}{Y_c^*(\lambda)}$  as a function of  $\lambda$  for a fixed  $f_x > 0$ .

Observe that the effects described above are nonmonotonic in the degree of openness. When  $f_x$  is small, only a small fraction of entrepreneur’s investment goes to cover fixed costs and the effect of increased  $\lambda$  is similar to that of a closed economy. When  $f_x$  is very high, the economy again behaves much like a closed economy because there is little trade as only the few most productive entrepreneurs find it worthwhile to enter exporting. In the intermediate range of fixed costs, their marginal reduction can thus have an ambiguous effect on sensitivity of the economy to credit frictions.<sup>21</sup>

The intuitions provided above do not give a full picture of the economy as they do not take into account the endogenous determination of wealth distribution and consequently the set of always constrained entrepreneurs, always unconstrained agents and those who switch as  $\lambda$  changes. However, I believe they capture the main mechanism of the model through which open economies are more sensitive to credit constraints than closed economies.

## 4 Extensions

In order to make sharp predictions, the benchmark model of this paper is based on some arguably strong assumptions. Distribution of ability across members of the same lineage is uncorrelated over time, intergenerational transfers are motivated by a simple “warm-glow” bequest motive and the interest rate is exogenously fixed by a “homogeneous” sector. The purpose of this section is to explore to what extent the basic insights of the model survive as those assumptions are relaxed. The following subsection briefly discusses the results of allowing for correlated ability while keeping the rest of the model intact. Subsection 4.2 numerically calibrates a model that uses a more standard infinite-lives framework, allows for correlated ability and endogenizes the interest rate.

### 4.1 Correlated Ability

So far the discussion has taken the entrepreneurial ability to be independently and identically distributed among members of a given lineage. In principle this can be problematic for a few reasons. It is reasonable to believe that entrepreneurial talents might be positively correlated among members of the same dynasty. This suggests that the dynasty might be able to save itself out of constraints: even if it starts poor but with high ability, in the presence of persistent ability shocks the dynasty is expected to accumulate wealth, possibly up to a point where the borrowing constraints are not binding. This changes the quantitative predictions of the model: other things equal, we would expect the losses from credit constraints to be smaller when ability is persistent.

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<sup>21</sup>A marginal reduction of variable transport costs  $\tau$  also in general has an ambiguous effect when  $f_x > 0$ .

This type of reasoning, applied to individual entrepreneurs or firms over time rather than dynasties, is at the core of recent contributions to the discussion of aggregate losses from credit frictions in a closed economy: see Moll (2009) and Midrigin and Xu (2010).

This level effect is not a great problem for this work, given that this paper is less interested in carefully quantifying the effects of credit constraints empirically. Here the main emphasis is on establishing qualitative differences of the impact of credit constraints on open and closed economies. The presence of persistence in ability by itself does not alter the prediction that credit constraints are relatively more important in an open economy with positive fixed costs. Recall that the higher sensitivity comes from the fact that constrained agents' revenues expand relatively more when they are exporters who have payed the fixed cost. But with the fixed-fraction-of-wealth bequest rule as in this paper, agents do not really care what the productivity of their descendants will be or whether they will be exporting or not. Because of that, it is not difficult to extend the model to the case in which entrepreneurial ability follows a Markov process. Appendix A.3.1 gives an example of an assumption under which one could reestablish all the analytical results of the paper (the complication being that now the endogenous object is the joint wealth-ability distribution). The key "higher sensitivity in the open economy" result is again obtained in all numerical simulations of this extended model.

The model with correlated ability has one feature that is worth mentioning. As productivity becomes more correlated, losses from credit constraints fall because for each ability level there is a level of wealth to which the dynasty would converge if it kept that productivity permanently, and with that wealth it would be unconstrained. Thus looking only in terms of aggregates (output of the differentiated sector, GDP, wealth) it might be difficult to distinguish an economy with good credit markets (high  $\lambda$ ) but little persistence in productivity and an economy with high degree of credit frictions (low  $\lambda$ ) but highly correlated productivity shocks. Both economies would have similar distribution of firm profits as well, as in either case few entrepreneurs would be credit constrained. However, the distributions of total income and wealth could be very different, depending on the underlying ability process.

## 4.2 Infinitely-Lived Agents

The assumption of warm-glow bequest motives with Cobb-Douglas specification of utility made the analysis of the stationary equilibrium of the economy tractable. It might be defensible if the model period is literally interpreted as a productive lifetime of a generation.<sup>22</sup> But any empirically oriented study of credit constraints needs to focus on data at a much higher frequency. When the model period is interpreted as a calendar year and subsequent "generations" as the same entrepreneur with stochastically evolving productivity, the idea that an entrepreneur simply saves a constant fraction of his end-of-year wealth regardless of his current productivity, wealth and degree of credit

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<sup>22</sup>Some authors argue that the warm-glow bequest motive is empirically more plausible than the perfect-altruism alternative, e.g. Andreoni (1989).

imperfections is not particularly plausible. In that case the standard macroeconomic framework with infinitely-lived agents might be a better approximation of reality.

This change of a modeling framework is an important check on the robustness of results coming from the benchmark model. One could expect that moving to the infinite-horizon planning problem can change not only quantitative but also qualitative properties of the solution. It is no longer clear that the effect of credit frictions will be higher in the open economy with fixed costs than in a closed economy. The reason is that the presence of fixed costs changes the savings behavior of forward-looking agents. When productivity is serially correlated, an entrepreneur that starts poor but has a good business idea has an incentive to postpone consumption and increase savings to relax his credit constraints in the future. When exporting is associated with fixed costs, the entrepreneur might save even more. With incentives to save higher in the open economy, it could happen that decreasing credit frictions is less important because the set of constrained entrepreneurs is in fact smaller. The answer might ultimately depend on parameters of the model and that is why in this section key parameters are calibrated to match some moments of the U.S. data. Before turning to the numerical investigation, I describe the alternative model in more detail, concentrating on the elements modified with respect to the benchmark model from previous sections.

### Statement of the Problem

Preferences of entrepreneurs now have the standard expected discounted utility form  $\mathbf{E}_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s})$ . Entrepreneur's ability follows the following Markov process: each period with probability  $1 - p_d$  productivity is carried over from the previous period. With probability  $p_d$  the agent receives a new ability drawn from a time-invariant distribution  $F$ . The draws are i.i.d. among agents receiving them in any period. In this simple formulation  $p_d$  is a (inverse) measure of persistence of productivity. One interpretation is that each business idea has a constant quality throughout its life but it becomes obsolete each period with probability  $p_d$ , in which case the entrepreneur has to come up with a new idea.<sup>23</sup>

There are two changes on the technological side of the economy. Firstly, capital now depreciates at a rate  $\delta$ . To keep notation consistent with earlier expressions  $r$  should now be understood as the rental cost of capital. Return  $r - \delta$  an entrepreneur gets from lending his wealth will be referred to as net interest rate. The second and more important difference is that the "homogeneous" sector is discarded: the interest rate will be endogenously determined to ensure equilibrium in the capital market. The reason for this change is that in an infinite-horizon problem the net interest rate has a much bigger effect on the economy than when bequests have the warm-glow form. Hence the results could be sensitive to the choice of interest rate if it was treated parametrically.

The production problem faced by an entrepreneur is unchanged. After observing his own pro-

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<sup>23</sup>Another interpretation, relating back to the earlier discussion in terms of generations, is that with probability  $p_d$  an entrepreneur dies and is replaced by his descendant, to whom he is perfectly altruistic. The descendant then runs the business to the best of his ability drawn from distribution  $F$ .

ductivity  $\theta$  and wealth  $w$  at the beginning of period and learning about the aggregate demand  $Y$ , capital cost  $r$  and credit conditions  $\lambda$  an entrepreneur makes the same investment and exporting decision as discussed earlier.<sup>24</sup> What does change in an important way is the decision of how to split the end-of-period wealth into current consumption and savings. Rather than being a constant fraction  $\alpha$  of wealth, the entrepreneur's savings now are a solution to the intertemporal optimization problem. In a stationary setting the entrepreneur's Bellman equation is as follows:

$$v(w, \theta) = \max_{c, w'} \{u(c) + \beta \mathbf{E} [v(w', \theta') | \theta]\} \quad (10)$$

subject to

$$c + w' \leq y(w, \theta) + (1 - \delta)w$$

$$c \geq 0, w' \geq 0$$

where  $y(w, \theta)$  is computed as in previous sections. Notice that the nonnegativity restriction on savings precludes borrowing for intertemporal consumption smoothing. This can only be restrictive with perfect credit markets. As long as  $\lambda$  is finite no entrepreneur would ever want to carry over debt to the next period as he would not be able to produce and consume from then on. There is no insurance market for entrepreneurial productivity.

The stationary equilibrium of this economy is defined in terms of joint wealth and ability distribution  $H(w, \theta)$ , output of the differentiated sector  $Y$ , rental cost of capital  $r$ , policy functions  $\{c(w, \theta), w'(w, \theta)\}$  such that:

1. Given  $Y$  and  $r$ ,  $\{c(w, \theta), w'(w, \theta)\}$  solve the entrepreneur's problem (10).
2. There is an equilibrium in the product market:

$$Y = \left[ L \int (\theta k_v(w, \theta))^{\frac{\gamma(\sigma-1)}{\sigma}} \left\{ 1 + I_X(w, \theta) \left[ (1 + \tau^{1-\sigma})^{\frac{1}{\sigma}} - 1 \right] \right\} H(dw, d\theta) \right]^{\frac{\sigma}{\sigma-1}},$$

where  $k_v$  denotes the variable capital input and  $I_X$  is the exporting indicator.

3. Capital market clears:

$$\int [k(w, \theta) - w] H(dw, d\theta) = 0.$$

4. The joint distribution of wealth and ability is a fixed point of mapping  $T^*$  induced by policy functions and exogenous ability process.

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<sup>24</sup>It is worth emphasizing that I do not consider the sunk costs of exporting. This is a stronger omission when the model is interpreted via lens of annual data. Incorporating sunk costs would add an additional state variable and complicate the exporting decision, making the comparison with the benchmark model less transparent.

Unlike in the benchmark case, I do not prove that the stationary equilibrium always exists and that it is unique. Like most of the similar literature, I merely verify in numerical simulations that the economy converges to the same equilibrium for different starting points.

## Calibration

The theoretical model of this paper was developed for the case of two symmetric countries. To keep the numerical exercises in the same spirit, in calibrating the parameters of the model I focus on U.S. data. Thinking of the US as a big country trading with another big country similar in economic size and development (say, the EU) should be a reasonable approximation for the purposes of this study.

The per-period utility function of entrepreneurs takes the logarithmic form:  $u(c) = \ln c$ . The distribution of new business ideas  $F$  has a finite support, taking 10 values on an equally spaced grid in the range  $[0.12, 3.6]$ . The probability mass distribution over those points corresponds to the Zipf distribution with parameter  $k_p$  truncated to the first 10 values.

Overall I have to specify 9 parameters of the model: time-preference rate  $\beta$ , elasticity of substitution  $\sigma$ , ability persistence parameter  $p_d$ , ability draw dispersion parameter  $k_p$ , degree of returns to scale in production  $\gamma$ , depreciation rate  $\delta$ , fixed and iceberg trade costs  $f_x$  and  $\tau$  and the degree of collateral constraints  $\lambda$ .

I choose  $\sigma = 5$  as the value for the elasticity of substitution between individual varieties produced by entrepreneurs. This number is in the mid range of estimates found in the extensive literature on the subject.<sup>25</sup> Variable transportation costs are fixed at  $\tau = 1.57$ . This choice together with  $\sigma = 5$  delivers 0.14 as a share of exports in total revenues among exporters, a value reported for the US by Bernard et al. (2007). Annual depreciation rate is set at  $\delta = 0.08$ . For the value of entrepreneurial ability persistence parameter I take  $p_d = 0.1$ . In one interpretation, this implies that the average useful life of a differentiated product is 10 years. This number is also close to 0.13 calibrated by Buera and Shin (2008) in a somewhat similar model and the implied autocorrelation of productivity is not far from 0.93 estimated by Midrigin and Xu (2010) for Korean firms.

The remaining five parameters are calibrated to five moments of the U.S. data. The technological parameter  $\gamma$  controlling the degree of returns to scale and the discount factor  $\beta$  both have a big influence on the capital-output ratio and equilibrium interest rate. Their calibration targets are therefore the capital-output ratio of 3.32 from Cooley and Prescott (1995) and the net real interest rate of 0.05 per annum. The fixed export costs  $f_x$  are chosen so that the share of exporters among businesses is 0.18, a number taken from Bernard et al. (2007). The parameter  $k_p$  governing the dispersion of productivity draws is set so that the Gini coefficient of income (comprising firm profits and interest income in the model) is 0.57, the value for the US calculated by Díaz-Giménez et al. (1997). Finally, the target for the credit frictions parameter  $\lambda$  is that ratio of borrowed capital to

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<sup>25</sup>See e.g. Broda and Weinstein (2006).

Table 1: Calibration Results

	Target	Model	Parameters
Capital-Output Ratio	3.32	3.71	$\gamma = 0.65$
Net Interest Rate	0.05	0.05	$\beta = 0.92$
Share of Exporters	0.18	0.18	$f_x = 0.81$
Income Gini	0.57	0.58	$k_p = 0.80$
External Finance/ GDP	1.33	1.33	$\lambda = 2.36$

output in the model matches the value of 1.33, calculated by Buera and Shin (2008) as the ratio of credit market liabilities to GDP in the US.

Table 1 summarizes the calibration exercise. The model tends to deliver  $\frac{K}{Y}$  ratio higher than in the data, all other targets are met almost exactly.

### Results: Level Effects

I now use the calibrated model to perform a number of counterfactual experiments. The main lesson from the theoretical model with fixed bequest rule is that credit constraints have a negative impact on aggregate outcomes of the economy as well as on individual welfare. The effects are relatively stronger in the open than in a closed economy, but only if exporting requires fixed costs. To see which results carry over to the current setting, I report below the effects of changing the degree of credit imperfections in parallel for three economies: in the benchmark calibrated economy with selection into exporting, in the economy closed to international trade but otherwise identical to the benchmark and in the open economy without fixed export costs. All comparisons relate to the stationary equilibria of the respective economies.

Not all effects from the economy with warm-glow bequests have their counterparts when agents are forward looking. It is no longer true that the invariant wealth distributions of economies with different  $\lambda$ 's (conditional on openness) can be ranked in terms of first order stochastic dominance, which is implied by Proposition 2. An increase in  $\lambda$  increases profits and with a fixed bequest rule also the savings of agents, shifting the wealth distribution to the right. But with infinite horizon planning, higher profits as a result of an increase in  $\lambda$  need not necessarily translate into higher savings. Observe that the interest rate in the calibrated economy is lower than the time-preference rate. Thus the least productive agents are generally dissaving: interest rate does not compensate for their impatience and for them future can only be better in terms of productivity. An improvement in the credit markets boosts their profits today but also in the future, lowering the minimum level of wealth they are willing to hold. Thus the stationary wealth distribution of the economy with higher  $\lambda$  can put more mass at low levels of wealth and this in fact occurs in the calibrated model.

Dominance of wealth distribution was a sufficient condition in Proposition 2 for the aggregate wealth and output to increase in  $\lambda$ . It is not necessary, however, as Figure 5 illustrates that both capital and output are still monotonically increasing in  $\lambda$ . In the benchmark case aggregate output

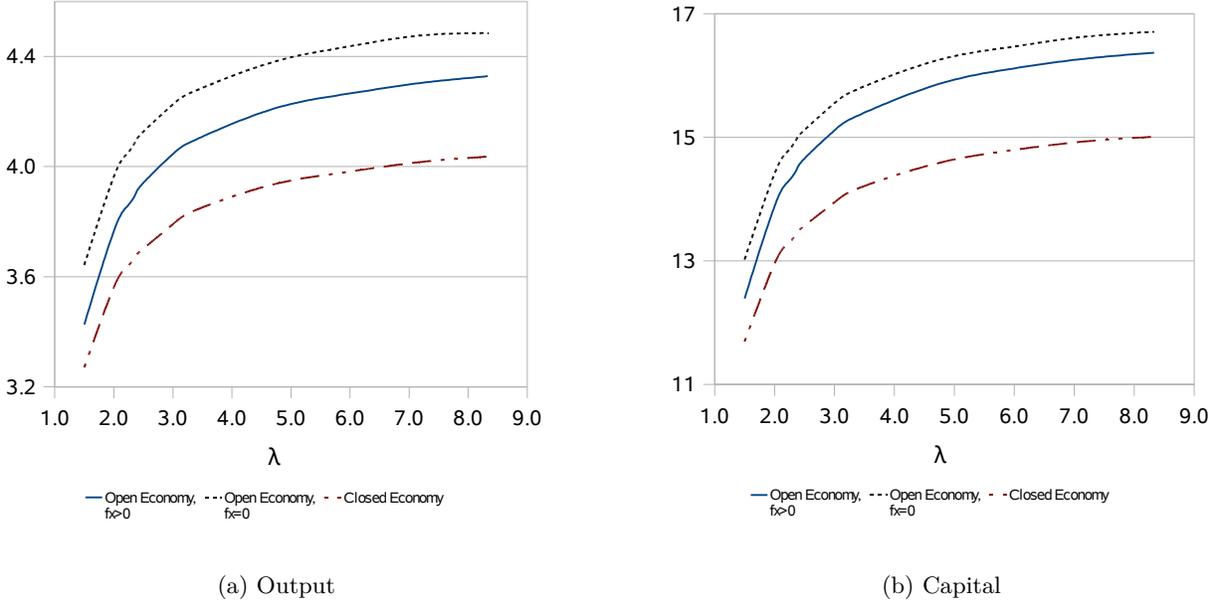


Figure 5: Output and Capital in Stationary Equilibrium as a Function of Credit Imperfections

is 14% lower and capital stock 16% lower than in a stationary equilibrium of economy without credit imperfections. A new feature is that the interest rate is also increasing with credit market performance. Intuitively, when credit frictions are mitigated the demand for capital rises because previously constrained agents want to borrow more while the incentives to save are diminished as agents see that they are less likely to be constrained in the future. The rise in the interest rate necessary to restore equilibrium is a general equilibrium effect that was absent in the model with interest rate fixed by the homogeneous sector.<sup>26</sup>

Comparing now open and closed economy equilibria for a fixed  $\lambda$ , the results in Proposition 3 appear robust to the change in modeling structure. In Figure 5 output and capital are higher in the open economy than in the closed economy at any level of credit market frictions. Evaluated at the calibrated level  $\lambda = 2.36$ , output is 6% higher and capital 8% higher in the benchmark economy than it would be in the corresponding autarky. If there were no fixed costs of exporting the figure would be 11% for both quantities. Moreover, it is true that the wealth (and joint wealth-ability) distribution in the trading economy first order stochastically dominates the closed economy distribution.

An interesting feature of the case with fixed exporting costs is the presence of discontinuities in entrepreneur's savings functions. With a fixed bequest rule,  $b(\cdot, \theta)$  is a continuous function of wealth

<sup>26</sup>When the interest rate is fixed exogenously at a too high level in a model with forward-looking agents there might be a perverse effect that increasing  $\lambda$  lowers capital stock. The reason is that agents keep extra capital just in case they need it when they are productive (and rent it at a high rate to the homogeneous sector when they are not), but that becomes unnecessary when  $\lambda$  is high.

albeit with a kink at the exporting wealth cutoff  $w_e(\theta)$  (for agents with  $\theta > \theta_{ue}$ ). In infinite-horizon problem there is no noticeable change in savings at  $w_e(\theta)$ . Instead there is a jump at wealth  $w_a$  such that  $w'(w_a, \theta) = w_e(\theta)$ . That is two entrepreneurs with identical ability and almost identical wealth (and hence income) make discretely different savings choices, with the slightly wealthier entrepreneur saving more towards the fixed cost he is likely to decide to pay next period.<sup>27</sup>

### Results: Openness and Sensitivity to Credit Constraints

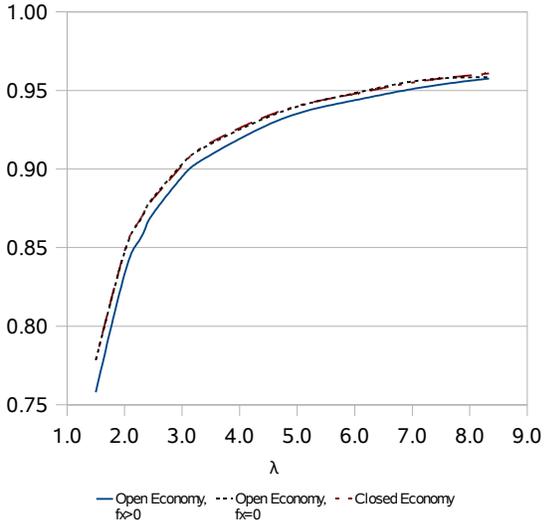
The main purpose of extending the stylized model from the first sections of the paper was to check the robustness of the finding that losses from credit frictions are larger in open economies subject to fixed exporting costs than either in closed economies or open economies without selection to exporting. Figure 6 illustrates the losses from credit imperfections (relative to no-frictions case  $\lambda = \infty$ ) in terms of aggregate output and capital in a stationary equilibrium. At any level of  $\lambda$  the ranking of losses is the same as in the simpler model. Quantitatively, the differences are not large: evaluated at the calibrated value  $\lambda = 2.36$  losses from credit imperfections translate into 13.8% output loss in the benchmark economy, compared to 12.6% loss in either autarky or hypothetical trading economy with  $f_x = 0$ . The qualitative pattern is nevertheless clearly present in the extended model as well. In particular, recall that the corollary of Proposition 4 was that without fixed costs of exporting aggregate losses in the open economy are exactly the same as in autarky. This is what can also be seen in Figure 6: the corresponding two lines are visually indistinguishable and the small differences can be attributed to the numerical imperfections of the computation.

Another way to assess the impact of openness on sensitivity to credit frictions is to look at the gains from trade (measured as an increase in output or capital relative to autarky) as a function of credit constraints. Numerical simulations such as those presented in Figure 4b found that GFT are increasing in  $\lambda$  when there is selection into exporting, while a corollary of Proposition 4 stated that GFT are independent of  $\lambda$  when exporting does not require fixed costs. Figure 7 illustrates the corresponding calculations for the extended model. While the relationships are not perfect - there is a small region with nonmonotonicity when  $f_x > 0$  and GFT fluctuate a little with  $f_x = 0$  - the general pattern that emerges is again congruent with the findings from the model with fixed bequests.

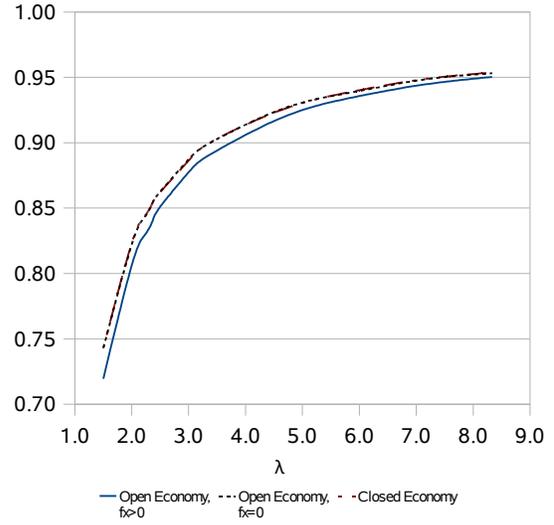
All the comparisons so far have used aggregate output or wealth. In the case of the benchmark model with utility linear in wealth concentrating on those quantities was justified by the fact that aggregate wealth was a measure of aggregate welfare. This is no longer true in the model with forward-looking entrepreneurs with logarithmic preferences. Thus the results for aggregates reported above do not immediately imply that analogous results (most notably higher sensitivity to credit frictions with fixed costs) hold for welfare comparisons. The best way to assess the impact of credit frictions on welfare would be to start with a stationary equilibrium with a particular  $\lambda$  and

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<sup>27</sup>There are more discontinuities of similar type, e.g. at  $w_{aa}$  s.t.  $w'(w_{aa}, \theta) = w_a$ , at  $w_{aij}$  s.t.  $w'(w_{aij}, \theta_i) = w_e(\theta_j)$  etc.



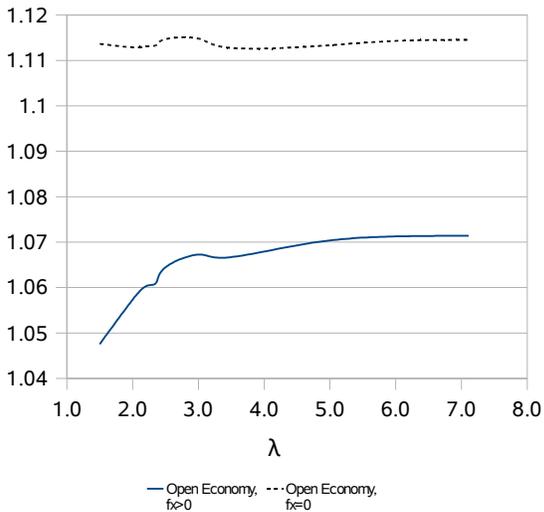
(a) Output



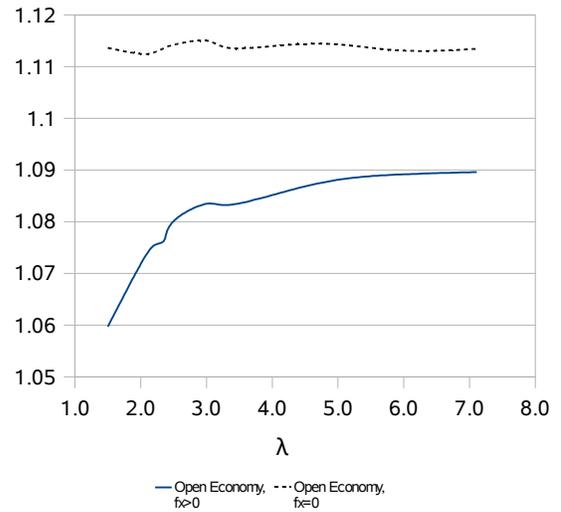
(b) Capital

Figure 6: Losses From Credit Frictions

Note: Output (panel a) and capital (panel b) in a stationary equilibrium with credit frictions  $\lambda$  relative to the case with perfect credit markets. A smaller number represents larger losses from credit frictions.



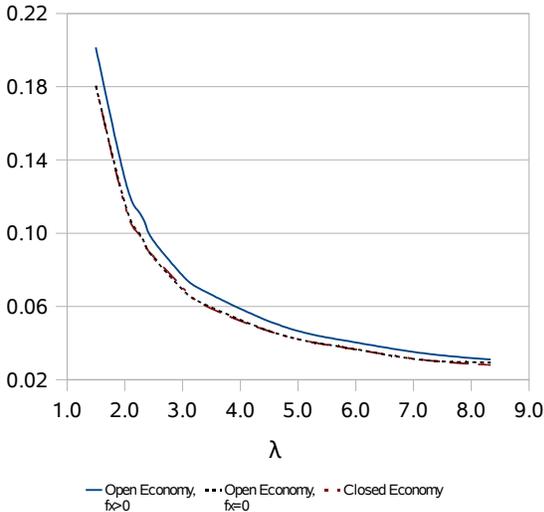
(a) Output



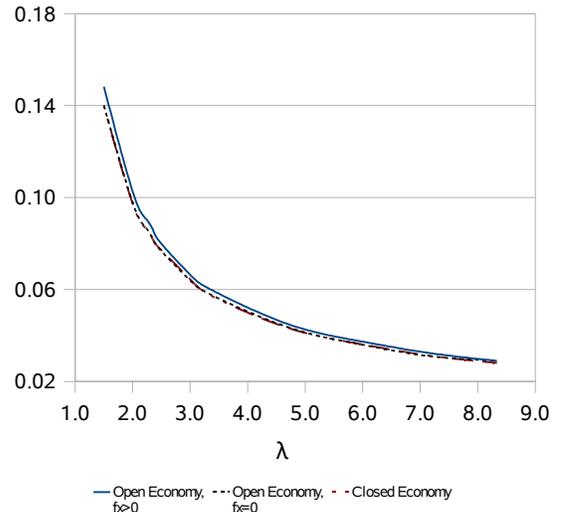
(b) Capital

Figure 7: Gains From Trade

Note: Gains from trade calculated as a ratio of output (panel a) and capital (panel b) in a stationary equilibria of open and closed economies with the same degree of credit frictions  $\lambda$ .



(a) Comparing Stationary Equilibria



(b) Comparing Using Initial Distribution

Figure 8: Welfare Gains From Reducing Credit Frictions

Note: A higher number represents bigger losses from credit frictions.

then calculate the gains from moving to a new stationary equilibrium with a higher  $\lambda'$ , taking into account the transitional dynamics. Unfortunately I do not provide such calculations as computing the transition paths between stationary equilibria in the extended model is not a trivial task. Instead I present the results of two simpler calculations. The first one simply compares welfare levels in the stationary equilibrium corresponding to a given  $\lambda$  and in the frictionless case. It is a result of the following thought experiment: suppose we could move the entire population of a country instantaneously to a frictionless equilibrium with its different wealth-ability distribution. How much in terms of a fraction of permanent consumption equivalent of utility in the new equilibrium would we have to take away from each agent so that the population is indifferent between staying in the old equilibrium and moving to the new one? In making that calculation I use a utilitarian social welfare function. More details can be found in Appendix A.3.2. As Figure 8a documents, the pattern emphasized throughout this paper is present also for this measure of welfare losses: they are higher in the open economy than in autarky, but only when there are fixed cost of entering the foreign market.

The second thought experiment is closer in spirit to the ideal calculation of welfare losses. Suppose the agents living in a country with credit constraints parametrized by  $\lambda$  were moved to a country with frictionless credit markets, but this time keeping their wealth and productivity. What is the fraction of permanent consumption equivalent of utility in the frictionless economy that if taken away from everybody makes the population indifferent between moving and staying? The answer is provided in Figure 8b. In the calibrated benchmark economy losses from credit constraints

calculated this way amount to 8.6% of permanent consumption equivalent of utility. Without fixed export costs or in autarky the corresponding number is 8.2%. The difference is not large, but yet again the same qualitative ranking emerges.

## 5 Conclusions

In this paper I develop a model of entrepreneurs who in their pursuit of economic activity at home and abroad encounter collateral constraints. I use the model to investigate whether the degree to which credit constraints affect the economy depends on openness to international trade. I find that in both open and closed economies credit frictions reduce aggregate performance of the economy and negatively affect individual welfare, but that those negative effects are stronger in open economies. This higher sensitivity to credit imperfections is present, however, only when entering the foreign market requires paying fixed costs, a case for which there is a strong empirical support. A basic intuition behind this result is that relaxing credit constraints leads to a relatively higher increase in working capital when an entrepreneur has used part of his borrowed funds to pay off the fixed cost first. The baseline model of this paper is quite stylized but the main insights derived from it survive in a calibration exercise involving a number of generalizations.

The findings of the paper depend on the assumption that entrepreneurs face constraints in financing both working capital and fixed outlays. However, the fact that all investment is treated symmetrically by lenders should not be important for the qualitative results. It is not immediately clear whether fixed costs should be treated as easier to collateralize or not in any case. To the extent that fixed costs represent purchase of fixed equipment they might be more difficult to abscond with, but when fixed exporting costs take the form of advertising campaign in a foreign country the associated funds might be difficult to monitor by lenders. All that matters in the end is that paying the fixed cost limits the maximum amount the entrepreneur can invest in working capital.

Throughout the paper I use the terminology of open and closed economies and I assume the fixed costs are used to enter the foreign market. It should be clear, however, that “openness” can be given a number of different interpretations. For example, essentially the same forces would be at work in a closed economy model where the fixed cost represents the cost of adopting a superior technology that increases entrepreneur’s operating profits.

Having studied the effects of fixed costs, the natural next step would be to incorporate also sunk costs into models with credit constraints. While the analysis would be more complicated due to the additional dimension of heterogeneity among firms, I expect that the extra non-convexity would only increase the aggregate losses from credit frictions. Allowing for a richer structure of costs can be important in quantifying the effects of credit frictions in future research.

## A Proofs

### A.1 Closed Economy

#### A.1.1 Net Capital Supply to the Homogeneous Sector

Without frictions, the capital demand by the differentiated sector is:

$$K^D = L \int_{\underline{\theta}}^{\bar{\theta}} \tilde{k}(\theta) dF(\theta) = L \left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right)^{\frac{-\sigma}{\sigma - \gamma(\sigma - 1)}} Y^{\frac{1}{\sigma - \gamma(\sigma - 1)}} \mathbf{E} \left( \theta^{\frac{\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} \right)$$

In the stationary setting, we can find the capital supply using the fact that the constant fraction  $\alpha$  of the end of period wealth is left as a bequest

$$\alpha [LE\tilde{\pi}(\theta) + (1 + r)K] = K$$

which gives

$$K = \frac{\alpha L}{1 - \alpha(1 + r)} \left[ \frac{\sigma - \gamma(\sigma - 1)}{\sigma} \right] Y^{\frac{1}{\sigma - \gamma(\sigma - 1)}} \left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right)^{\frac{-\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} \mathbf{E} \left( \theta^{\frac{\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} \right)$$

Setting  $K \geq K^D$  we obtain

$$\mathbf{Assumption 1.} \quad \frac{\alpha}{1 - \alpha(1 + r)} \left[ \frac{\sigma - \gamma(\sigma - 1)}{\sigma} \right] \left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right) \geq 1$$

#### A.1.2 Existence and Uniqueness of the Stationary Equilibrium

In this section of the Appendix I prove results for i.i.d. distributed ability shocks as in Sections 2-3. Specifically, I make the following assumption:

**Assumption 2.** *Entrepreneurial ability shocks  $\theta$  are distributed i.i.d. over time within a lineage and across lineages at any time. The distribution of shocks is summarized by the probability space  $(Z, \mathcal{Z}, \mathcal{F})$  where  $Z = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$  and  $\mathcal{Z}$  is the Borel  $\sigma$ -algebra on  $Z$ .  $\mathcal{F}$  has the following property: for some  $\omega > 0$*

$$\mathcal{F}((a, b]) \geq \omega(b - a), \quad \forall \underline{\theta} \leq a \leq b \leq \bar{\theta}$$

I use  $F$  to denote the distribution function corresponding to the probability measure  $\mathcal{F}$ . Assumption 2 is satisfied, for example, by any continuous distribution with positive density on  $Z$  such as a truncated Pareto distribution.

To show the results of interest I first establish a sequence of intermediary lemmas.

**Definition 1.** Let  $P : W \times W \rightarrow [0, 1]$  be a transition function describing a Markov process. A stationary (or invariant) distribution  $\mathcal{G}$  of a Markov process corresponding to  $P$  is a fixed point of the mapping  $T^*$  defined as follows: for any probability measure  $\mu$  on  $(W, \mathcal{W})$

$$T^*\mu(B) = \int P(s, B) \mu(ds) \text{ for all } B \in \mathcal{W}$$

I start by showing the existence of a unique stationary distribution over wealth for a given lineage. This distribution is also the cross-sectional wealth distribution in the stationary equilibrium, since ability draws are distributed i.i.d. across lineages and there is a continuum of lineages.

**Lemma 1.** *For any given  $Y > 0$  and  $\lambda$  there exists a unique stationary distribution  $\mathcal{G}$  over  $(W, \mathcal{W})$ .*

*Proof.* The proof is divided into two steps. First I show that there exists an ergodic set  $W = [\underline{w}, \bar{w}]$  such that if an agent receives bequests  $w \in W$ , the bequest in his lineage never leaves  $W$ . Then I show that the conditions of Theorem 2 in Hopenhayn and Prescott (1992) are satisfied, in particular, that the transition function  $P$  induced by the bequest rule  $b(w, \theta)$  and distribution of ability shocks is increasing and satisfies the Monotone Mixing Condition.

To prove the existence of the ergodic set notice that  $b(w, \theta)$  is increasing and continuous in both arguments. Moreover,  $\lim_{w \rightarrow \infty} \frac{\partial}{\partial w} b(w, \theta) = \alpha(1+r) < 1$  which is implied by Assumption 1 and  $\lim_{w \rightarrow 0} \frac{\partial}{\partial w} b(w, \theta) = \infty$  for all  $\theta > 0$ . Thus the lower and upper bound of the ergodic set are found from  $b(\underline{w}, \underline{\theta}) = \underline{w}$  and  $b(\bar{w}, \bar{\theta}) = \bar{w}$ , respectively. In addition, it can be verified that Assumption 1 guarantees that  $\bar{w} > \tilde{w}(\bar{\theta})$  and  $\underline{w} > \tilde{w}(\underline{\theta})$  so that both least able and poorest as well as most able and richest agents are not credit constrained.

As a consequence, we can provide explicit expressions for the bounds of the ergodic set:

$$\bar{w} = \frac{\alpha}{1 - \alpha(1+r)} \tilde{\pi}(\bar{\theta}) = \frac{\alpha}{1 - \alpha(1+r)} Y^{\frac{1}{[\sigma - \gamma(\sigma - 1)]}} \bar{\theta}^{\frac{\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} \left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right)^{\frac{-\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} \left[ \frac{\sigma - \gamma(\sigma - 1)}{\sigma} \right] \quad (11)$$

$$\underline{w} = \frac{\alpha}{1 - \alpha(1+r)} \tilde{\pi}(\underline{\theta}) = \frac{\alpha}{1 - \alpha(1+r)} Y^{\frac{1}{[\sigma - \gamma(\sigma - 1)]}} \underline{\theta}^{\frac{\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} \left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right)^{\frac{-\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} \left[ \frac{\sigma - \gamma(\sigma - 1)}{\sigma} \right] \quad (12)$$

Given  $W = [\underline{w}, \bar{w}]$ , let  $\mathcal{W}$  denote the Borel  $\sigma$ -algebra on  $W$ . With  $\theta$  an i.i.d. stochastic process, it is a standard argument that the bequest process is a stationary Markov process with corresponding transition function  $W \times \mathcal{W} \rightarrow [0, 1]$  given by:

$$P(w, B) = \mathcal{F} \{ \theta : b(w, \theta) \in B \}, \text{ for all } B \in \mathcal{W}$$

Now I show that  $P$  is monotone, in the following sense:  $w, w' \in W$  and  $w' \geq w$  implies  $P(w', \cdot) \succeq P(w, \cdot)$  (first-order stochastic dominance). Since to establish f.s.d. it is enough to look at monotone sets we can restate the conditions as:  $\forall w, w' \in W, w' \geq w$  implies that  $\forall x \in W, P(w', [x, \bar{w}]) \geq P(w, [x, \bar{w}])$ .

To see why this is the case let  $B_w = \{ \theta : b(w, \theta) \in [x, \bar{w}] \}$ . Define  $B_{w'}$  analogously. Pick  $\theta_1 \in B_w$ , so  $b(w, \theta_1) \in [x, \bar{w}]$ . But since  $b$  is increasing in the first argument,  $b(w', \theta_1) \in [x, \bar{w}]$  as well so  $\theta_1 \in B_{w'}$ . Hence  $B_w \subset B_{w'}$ . So  $P(w', [x, \bar{w}]) = \mathcal{F}(B_{w'}) \geq \mathcal{F}(B_w) = P(w, [x, \bar{w}])$ .

Now I show that  $P$  satisfies the following Monotone Mixing Condition:

**Condition.** (MMC): There exists  $w^* \in W$  and  $N \geq 1$  such that  $P^N(\underline{w}, [w^*, \bar{w}]) > 0$  and  $P^N(\bar{w}, [\underline{w}, w^*]) > 0$ .

Let  $w^* = \frac{\underline{w} + \bar{w}}{2}$  and let  $\theta^*$  be such that  $b(w^*, \theta^*) = w^*$ . Since  $b$  is increasing in both arguments we know that such  $\theta^*$  with  $\underline{\theta} < \theta^* < \bar{\theta}$  exists. Now let  $\varepsilon_1 = \frac{\bar{\theta} - \theta^*}{2} > 0$ . Then  $b(w, \theta) - w$  is strictly positive and continuous on  $[\underline{w}, w^*]$  for all  $\bar{\theta} \geq \theta \geq \theta^* + \varepsilon_1$  so it is uniformly bounded below by some

$\varepsilon_2 > 0$ . Thus there exists integer  $N_1$  such that  $N \geq N_1 \Rightarrow b^N(\underline{w}, \theta^* + \varepsilon_1) > w^*$  where  $b^N$  is the  $N$ -th iterate of  $b(\cdot, \theta^* + \varepsilon_1)$ . Similarly let  $\varepsilon_3 = \frac{\theta^* - \underline{\theta}}{2} > 0$ . Then  $w - b(w, \theta)$  is strictly positive and continuous on  $[w^*, \bar{w}]$  for all  $\theta \leq \theta \leq \theta^* - \varepsilon_3$  so it is uniformly bounded below by some  $\varepsilon_4 > 0$ . Thus there exists integer  $N_2$  such that  $N \geq N_2 \Rightarrow b^N(\bar{w}, \theta^* - \varepsilon_3) < w^*$  where  $b^N$  is the  $N$ -th iterate of  $b(\cdot, \theta^* - \varepsilon_3)$ .

Now let  $N = \max\{N_1, N_2\}$  then  $P^N(\underline{w}, [w^*, \bar{w}]) \geq (\mathcal{F}[\theta^* + \varepsilon_1, \bar{\theta}])^N \geq [\omega(\bar{\theta} - \theta^* - \varepsilon_1)]^N > 0$  and  $P^N(\bar{w}, [\underline{w}, w^*]) \geq (\mathcal{F}[\underline{\theta}, \theta^* - \varepsilon_3])^N \geq (\omega(\theta^* - \varepsilon_3 - \underline{\theta})) > 0$  where we have used Assumption 2. This proves that  $P$  satisfies MMC.

Since  $P$  is increasing and satisfies the MMC<sup>28</sup> by Theorem 2 in Hopenhayn and Prescott (1992) it has a unique invariant distribution  $\mathcal{G}$  and for any initial distribution  $\mathcal{G}_0$ ,  $T^{*n}\mathcal{G}_0 \Rightarrow \mathcal{G}$ .  $\square$

The next lemma shows the relationship between invariant distributions obtained for different levels of aggregate demand holding the degree of market imperfections constant.

**Lemma 2.** *Let  $\mathcal{G}_1$  be the invariant measure corresponding to  $(Y_1, \lambda)$  and define  $\mathcal{G}_2$  similarly for  $(Y_2, \lambda)$ . Let  $\phi(w) = \frac{1}{t}w$  where  $t = \left(\frac{Y_1}{Y_2}\right)^{\frac{1}{\sigma - \gamma(\sigma - 1)}}$ . Then  $\mathcal{G}_2(A) = \mathcal{G}_1(\phi^{-1}(A))$  for all  $A \in \mathcal{W}_2$ .*

*Proof.* Since  $\mathcal{G}_1$  is the invariant probability measure for  $(Y_1, \lambda)$  it satisfies:

$$\mathcal{G}_1(B) = \int_{W_1} P_1(s, B) \mathcal{G}_1(ds) \text{ for all } B \in \mathcal{W}_1$$

We want to show that  $\mathcal{G}_2$  given as:

$$\mathcal{G}_2(A) = \mathcal{G}_1(\phi^{-1}(A)) \quad \forall A \in \mathcal{W}_2$$

is an invariant measure for  $(Y_2, \lambda)$ . It is enough to consider increasing sets of the form  $A = [x, \bar{w}_2]$ . Fixed one such set,  $A$ . Observe that  $\phi : W_1 \rightarrow W_2$  is measurable as it is continuous. Moreover, because  $\underline{w}_2 = \frac{1}{t}\underline{w}_1$  and  $\bar{w}_2 = \frac{1}{t}\bar{w}_1$  we have  $\phi(W_1) = W_2$ . Since  $P_2$  is a transition function,  $P_2(\cdot, A)$  is  $\mathcal{W}_2$ -measurable. An application of a general change of variable theorem then gives us:

$$\int_{W_1} P_2(\phi(s), A) \mathcal{G}_1(ds) = \int_{W_2} P_2(s, A) \mathcal{G}_2(ds)$$

where  $\mathcal{G}_2$  is as defined above. For  $\mathcal{G}_2$  to be an invariant measure, the expression above must equal  $\mathcal{G}_2(A)$ . Thus on the one hand we must have

$$\mathcal{G}_2(A) = \int_{W_1} P_2(\phi(s), A) \mathcal{G}_1(ds)$$

but since  $\mathcal{G}_1$  is an invariant measure also

$$\mathcal{G}_2(A) = \mathcal{G}_1(\phi^{-1}(A)) = \int_{W_1} P_1(s, \phi^{-1}(A)) \mathcal{G}_1(ds)$$

To establish the claim it is sufficient to show that  $P_1(s, \phi^{-1}(A)) = P_2(\phi(s), A) \quad \forall s \in W_1$ . That is to show that  $\forall s \in W_1$ :

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<sup>28</sup>Other conditions of the theorem are trivially satisfied here.

$$\mathcal{F}(\theta : b_1(s, \theta) \in [tx, t\bar{w}_2]) = \mathcal{F}\left(\theta : b_2\left(\frac{1}{t}s, \theta\right) \in [x, \bar{w}_2]\right)$$

A sufficient condition is that  $b_1(s, \theta) = tb_2\left(\frac{1}{t}s, \theta\right)$  for all  $s \in W_1$  or equivalently  $b_1(ta, \theta) = tb_2(a, \theta)$  for all  $a \in W_2$ . This equality can be directly verified using the expressions for  $b$  function. The tedious details are omitted here.  $\square$

Sometimes it will be convenient to refer to distribution functions rather than probability measures. Given a measure  $\mathcal{G}$  on  $(W, \mathcal{W})$  let  $\hat{\mathcal{G}}$  be its extension to  $(\mathbb{R}, \mathcal{B})$ . Then we can define a distribution function  $G$  corresponding to  $\mathcal{G}$  via  $G(w) = \hat{\mathcal{G}}((-\infty, w])$ . Lemma 2 has the following immediate corollary:

**Corollary 1.** *Let  $G_1$  be a distribution function for the invariant measure corresponding to  $(Y_1, \lambda)$  and define  $G_2$  similarly for  $(Y_2, \lambda)$ . Then  $G_2(w) = G_1(tw)$  for all  $w \in \mathbb{R}$ , where  $t = \left(\frac{Y_1}{Y_2}\right)^{\frac{1}{\sigma-\gamma(\sigma-1)}}$ .*

An immediate consequence of Corollary 1 is that if  $Y_2 \geq Y_1$  then  $\mathcal{G}_2 \succeq \mathcal{G}_1$ .

By Lemma 1, for a given  $(\lambda, Y_0)$  pair the wealth distribution converges to a unique invariant distribution  $G$ . That is if agents take the aggregate expenditure on differentiated good as given at  $Y_0$ , the economy with credit constraints parametrized by  $\lambda$  will converge to  $G$ . The resulting aggregate output  $Y_1$  will in general be different from  $Y_0$ , however. The stationary equilibrium of the economy will have aggregate output  $Y^*$  that is a fixed point of the mapping  $Y_1 = \zeta(\lambda, Y_0)$ , so that  $Y^* = \zeta(\lambda, Y^*)$  where

$$\zeta(\lambda, Y) = \left[ L \int \int [\theta k(w, \theta; \lambda, Y)]^{\gamma \frac{\sigma-1}{\sigma}} dG(w; \lambda, Y) dF(\theta) \right]^{\frac{\sigma}{\sigma-1}}$$

where it has been made explicit that the investment  $k$  and wealth distribution  $G$  depend parametrically on  $(\lambda, Y)$ . The following lemma establishes an important property of the  $\zeta$  function:

**Lemma 3.**  $\frac{\zeta(\lambda, Y_2)}{\zeta(\lambda, Y_1)} = t^{-\gamma}$  where  $t = \left(\frac{Y_1}{Y_2}\right)^{\frac{1}{\sigma-\gamma(\sigma-1)}}$ .

*Proof.* Lemma 3 follows from direct computation using expressions for  $k$ , bounds of the ergodic set (11)-(12), Corollary 1 and simple change of variables.  $\square$

**Corollary 2.**  $\zeta(\lambda, Y) = c(\lambda) Y^{\frac{\gamma}{\sigma-\gamma(\sigma-1)}}$  where  $c(\lambda) \equiv \zeta(\lambda, 1)$ .

The expression for  $\zeta$  given in the corollary will be very useful. Now we are in position to prove:

**Proposition** (1, reminded). *For any  $\lambda \geq 1$  there exists a unique stationary equilibrium of the economy.*

*Proof.* By corollary 2 there exists a unique fixed point of the  $\zeta$  function given by:

$$Y^*(\lambda) = c(\lambda) \frac{\sigma-\gamma(\sigma-1)}{\sigma(1-\gamma)} \tag{13}$$

$Y^*(\lambda)$  is the equilibrium demand for the differentiated products and  $\mathcal{G}(\cdot; \lambda, Y^*(\lambda))$  is the stationary wealth measure in the economy.  $\square$

### A.1.3 Comparative Statics with $\lambda$

The following lemma is important for comparative statics with  $\lambda$ .

**Lemma 4.** *Suppose  $\lambda_2 > \lambda_1$ . Then the invariant wealth measure corresponding to  $(\lambda_2, Y)$  first order stochastically dominates the invariant measure corresponding to  $(\lambda_1, Y)$ :  $\mathcal{G}_{\lambda_2} \succeq \mathcal{G}_{\lambda_1}$ .*

*Proof.* Without loss of generality fix  $Y = 1$ . I will show that  $T_{\lambda_2}^*$  dominates  $T_{\lambda_1}^*$ . Since the mapping  $T_{\lambda_i}^*$  is increasing (because  $P_{\lambda_i}$  is increasing) the result follows from Corollary 3 in Hopenhayn and Prescott (1992).

First observe that the ergodic set  $W$  is determined by unconstrained agents so changing  $\lambda$  does not affect the ergodic set. I need to prove that for any probability measure  $\mu$  on  $(W, \mathcal{W})$ ,  $T_{\lambda_2}^* \mu \succeq T_{\lambda_1}^* \mu$ . Let  $\mu$  be any such measure. It's enough to show the claim for all increasing sets  $A \subset W$ . Thus I have to show that

$$T_{\lambda_2}^* \mu(A) = \int P_{\lambda_2}(s, A) \mu(ds) \geq \int P_{\lambda_1}(s, A) \mu(ds) = T_{\lambda_1}^* \mu(A) \text{ for all increasing } A \subset W$$

Clearly it's enough to show that  $P_{\lambda_2}(s, A) \geq P_{\lambda_1}(s, A)$  for all  $s \in W$ . Since increasing sets have the form  $[w, \bar{w}]$  the previous inequality is equivalent to  $P_{\lambda_2}(s, [w, \bar{w}]) \geq P_{\lambda_1}(s, [w, \bar{w}])$  or  $\mathcal{F}(\theta : b_{\lambda_2}(s, \theta) \geq w) \geq \mathcal{F}(\theta : b_{\lambda_1}(s, \theta) \geq w)$  for all  $s \in W$ , all  $w \in W$ . This is true because  $b_{\lambda_2}(s, \theta) \geq b_{\lambda_1}(s, \theta)$ ,  $\forall s \in W$ .  $\square$

Using the previous lemma it's easy to show the next result:

**Lemma 5.** *If  $\lambda_2 > \lambda_1$  then  $c(\lambda_2) \geq c(\lambda_1)$ . The inequality is strict if  $\lambda_1 < \lambda_*$ .*

*Proof.* Let  $k_{\lambda_1}$  be investment function for  $\lambda_1$  and define  $k_{\lambda_2}$  similarly. Then we have:

$$\begin{aligned} c(\lambda_1) &= \left[ L \int \int [\theta k_{\lambda_1}(w, \theta; \lambda, 1)]^{\frac{\sigma-1}{\sigma}} dG(w; \lambda_1, 1) dF(\theta) \right]^{\frac{\sigma}{\sigma-1}} \\ &\leq \left[ L \int \int [\theta k_{\lambda_2}(w, \theta; \lambda, 1)]^{\frac{\sigma-1}{\sigma}} dG(w; \lambda_1, 1) dF(\theta) \right]^{\frac{\sigma}{\sigma-1}} \\ &\leq \left[ L \int \int [\theta k_{\lambda_2}(w, \theta; \lambda, 1)]^{\frac{\sigma-1}{\sigma}} dG(w; \lambda_2, 1) dF(\theta) \right]^{\frac{\sigma}{\sigma-1}} = c(\lambda_2) \end{aligned}$$

where the first inequality follows because  $k_{\lambda_2}(w, \theta) \geq k_{\lambda_1}(w, \theta)$  for all  $w$  and all  $\theta$ , and the second inequality follows because  $k_{\lambda_2}(\cdot, \theta)$  is nondecreasing and  $\mathcal{G}_{\lambda_2} \succeq \mathcal{G}_{\lambda_1}$  by Lemma 4. Moreover, if there are any (positive measure) agents that are constrained in the invariant distribution for  $\lambda_1$ , then  $k_{\lambda_2}(w, \theta) > k_{\lambda_1}(w, \theta)$  for those agents and as a result we have a strict inequality in the derivation above. Most productive but poorest agent will be constrained if  $\underline{w} < \tilde{w}(\bar{\theta})$  or if

$$\frac{\alpha}{1 - \alpha(1 + r)} \left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right) \left[ \frac{\sigma - \gamma(\sigma - 1)}{\sigma} \right] < \frac{1}{\lambda} \left( \frac{\bar{\theta}}{\underline{\theta}} \right)^{\frac{\gamma(\sigma-1)}{\sigma-\gamma(\sigma-1)}}$$

Let  $\lambda_*^1$  be  $\lambda$  solving the above condition with equality and define  $\lambda_* = \max\{\lambda_*^1, 1\}$ . Then if  $\lambda_1 < \lambda_*$  the  $(\underline{w}, \bar{\theta})$  entrepreneurs will be constrained. If there is a nonzero measure of them the result

follows. Otherwise we can set  $\lambda_* - \varepsilon$  for arbitrarily small  $\varepsilon$  and the MMC and continuity of  $k$  will guarantee that there will be a positive measure of constrained agents.  $\square$

Assembling the various intermediate results, we get the main comparative statics result of this section:

**Proposition** (2, reminded). *If  $\lambda_2 > \lambda_1$  then the wealth distribution in the stationary equilibrium of the economy with  $\lambda = \lambda_2$  first order stochastically dominates the distribution in the stationary equilibrium of the economy with  $\lambda = \lambda_1$ . Aggregate output of the differentiated sector and aggregate capital are (weakly) higher in the economy with better credit markets:  $Y^*(\lambda_2) \geq Y^*(\lambda_1)$ ,  $K(\lambda_2) \geq K(\lambda_1)$ .*

*Proof.* From the proof of Proposition 1 we have  $\frac{Y^*(\lambda_2)}{Y^*(\lambda_1)} = \left[ \frac{c(\lambda_2)}{c(\lambda_1)} \right]^{\frac{\sigma - \gamma(\sigma - 1)}{\sigma(1 - \gamma)}} \geq 1$  where the inequality follows from Lemma 5. An implication of Corollary 1 is that  $\mathcal{G}(\cdot; \lambda, Y_2) \succeq \mathcal{G}(\cdot; \lambda, Y_1)$  for  $Y_2 \geq Y_1$ . Combining this with Lemma 4 we get  $\mathcal{G}(\cdot; \lambda_2, Y^*(\lambda_2)) \succeq \mathcal{G}(\cdot; \lambda_2, Y^*(\lambda_1)) \succeq \mathcal{G}(\cdot; \lambda_1, Y^*(\lambda_1))$ . It follows immediately that  $K(\lambda_2) = \int w \mathcal{G}(dw; \lambda_2, Y^*(\lambda_2)) \geq \int w \mathcal{G}(dw; \lambda_1, Y^*(\lambda_1)) = K(\lambda_1)$ .  $\square$

## A.2 Open Economy

### A.2.1 Net Capital Supply to the Homogeneous Sector

With net capital supply to the homogeneous sector, output of the differentiated sector in the open economy without frictions is given by:

$$Y = \left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right)^{\frac{-\sigma\gamma}{\sigma(1-\gamma)}} \left\{ L^{\frac{\sigma}{\sigma-1}} \left[ \mathbf{E} \left( \left\{ 1 + \tilde{I}_X(\theta) \left[ (1 + \tau^{1-\sigma})^{\frac{1}{\phi}} - 1 \right] \right\} \theta^{\frac{\gamma(\sigma-1)}{\phi}} \right) \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\frac{\phi}{\sigma(1-\gamma)}} \quad (14)$$

where  $\tilde{I}_X(\theta) = \begin{cases} 1 & \text{if } \theta \geq \theta_{ue} \\ 0 & \text{o/w} \end{cases}$  is an exporting indicator.

To ensure nonnegative capital supply to the homogeneous sector in the stationary equilibrium of an open economy without frictions Assumption 1 has to be strengthened to:

**Assumption 3.**

$$\left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right)^{\frac{-\sigma}{\sigma - \gamma(\sigma - 1)}} \frac{1 - \alpha(1 + r)}{\alpha} \leq \left\{ \left[ \frac{\sigma - \gamma(\sigma - 1)}{\sigma} \right] \left( \frac{1}{\gamma} \frac{\sigma}{\sigma - 1} r \right)^{\frac{-\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} - f_x r \frac{\mathbf{E} \left[ \tilde{I}_X(\theta) \right]}{E \left( \left\{ 1 + \tilde{I}_X(\theta) \left[ (1 + \tau^{1-\sigma})^{\frac{1}{\sigma - \gamma(\sigma - 1)}} - 1 \right] \right\} \theta^{\frac{\gamma(\sigma - 1)}{\sigma - \gamma(\sigma - 1)}} \right)} \right\}$$

Notice that if  $f_x = 0$  then this collapses just to Assumption 1.

### A.2.2 Exporters and Nonexporters with Collateral Constraints

In this subsection I derive the cutoff level of wealth required to induce an entrepreneur to become an exporter. First consider agents that are constrained even if they produce for domestic market only, i.e. those with  $w < \frac{1}{\lambda} \tilde{k}_H(\theta)$ . Producing for domestic market only yields them profits

$$(\theta \lambda w)^{\gamma \frac{\sigma-1}{\sigma}} Y^{\frac{1}{\sigma}} - r \lambda w$$

If they decide to export, they invest up to the borrowing limit obtaining profit

$$\left( \theta \left( \lambda w - f Y^{\frac{1}{\sigma-\gamma(\sigma-1)}} \right) \right)^{\gamma \frac{\sigma-1}{\sigma}} Y^{\frac{1}{\sigma}} \left( 1 + \tau^{1-\sigma} \right)^{\frac{1}{\sigma}} - \lambda w r \quad (15)$$

Comparing those two expressions we find that exporting is more profitable when

$$w \geq \frac{f}{\lambda} Y^{\frac{1}{\sigma-\gamma(\sigma-1)}} \frac{(1 + \tau^{1-\sigma})^{\frac{1}{\gamma(\sigma-1)}}}{\left[ (1 + \tau^{1-\sigma})^{\frac{1}{\gamma(\sigma-1)}} - 1 \right]} \equiv w_e^0$$

Notice that  $w_e^0$  does not depend on  $\theta$  while  $\frac{1}{\lambda} \tilde{k}_H(\theta)$  is increasing in ability. As long as the support of the distribution  $[\underline{\theta}, \bar{\theta}]$  is wide enough, there exists productivity level  $\theta_{pde}$  (for poor indifferent between domestic-only and exporting) such that  $\frac{1}{\lambda} \tilde{k}_H(\theta_{pde}) = w_e^0$ . Some algebra reveals that is given by

$$\theta_{pde} = \theta_{ue} \left\{ \frac{(1 + \tau^{1-\sigma})^{\frac{1}{\gamma(\sigma-1)}}}{\left[ (1 + \tau^{1-\sigma})^{\frac{1}{\gamma(\sigma-1)}} - 1 \right]} \left\{ (1 + \tau^{1-\sigma})^{\frac{1}{\sigma-\gamma(\sigma-1)}} - 1 \right\} \left[ \frac{\sigma - \gamma(\sigma - 1)}{\gamma(\sigma - 1)} \right] \right\}^{\frac{\sigma-\gamma(\sigma-1)}{\gamma(\sigma-1)}}$$

The exporting decision described above (export if  $w \geq w_e^0$ ) was derived under the assumption that the entrepreneur is constrained on the domestic market and as such applies only to entrepreneurs with ability above  $\theta_{pde}$ . Those with lower ability would be unconstrained on the domestic market with wealth  $w_e^0$  and for them we need a separate analysis.

Consider the situation of entrepreneurs with  $\frac{1}{\lambda} k_H(\theta) \leq w < \tilde{w}(\theta)$ . We want to find when it is optimal to invest  $k_H(\theta)$  and when it pays to become and exporter. The first option gives net profits  $\tilde{\pi}_H(\theta)$ . Since the agent is constrained in that he cannot borrow enough to produce his first best for both markets, he will go to his borrowing limit if he becomes exporter getting profit as in (15). The cutoff wealth  $w_e^1(\theta)$  that makes an agent indifferent between exporting or not is given implicitly as a solution<sup>29</sup> to the following equation:

$$\begin{aligned} & \left( \theta \left( \lambda w_e^1(\theta) - f Y^{\frac{1}{\sigma-\gamma(\sigma-1)}} \right) \right)^{\gamma \frac{\sigma-1}{\sigma}} Y^{\frac{1}{\sigma}} \left( 1 + \tau^{1-\sigma} \right)^{\frac{1}{\sigma}} - \lambda w_e^1(\theta) r \\ & - Y^{\frac{1}{[\sigma-\gamma(\sigma-1)]}} \theta^{\frac{\gamma(\sigma-1)}{\sigma-\gamma(\sigma-1)}} \left( \frac{1}{\gamma} \frac{\sigma}{\sigma-1} r \right)^{\frac{-\gamma(\sigma-1)}{\sigma-\gamma(\sigma-1)}} \left[ \frac{\sigma - \gamma(\sigma - 1)}{\sigma} \right] = 0 \end{aligned}$$

By construction we have  $w_e^1(\theta_{ue}) = \tilde{w}(\theta_{ue})$ . Moreover, it can be verified that  $w_e^1(\theta_{pde}) = w_e^0$ . Combining those results we see that the partition of entrepreneurs in the  $\theta - w$  space into constrained/unconstrained exporters/nonexporters is as shown in Figure 2.

<sup>29</sup>To be precise it is the smaller root. If we force agents to invest too much and export, their profit will eventually be lower than just optimally investing in home production, even though exporting is better for  $w$  in the relevant range.

### A.2.3 Properties of the Stationary Distribution

Adapting the results from the closed to the open economy requires only minor modifications. Because exporting opportunities increase maximum attainable level of profits, for any fixed  $Y$  the ergodic set  $W = [\underline{w}, \bar{w}]$  has to be modified from (11)-(12) to:

$$\bar{w} = \frac{\alpha}{1 - \alpha(1+r)} \left[ (1 + \tau^{1-\sigma})^{\frac{1}{\sigma-\gamma(\sigma-1)}} \tilde{\pi}_H(\bar{\theta}) + \left( w - f_x Y^{\frac{1}{\sigma-\gamma(\sigma-1)}} \right) r \right] \quad (16)$$

$$\underline{w} = \frac{\alpha}{1 - \alpha(1+r)} \left\{ \tilde{I}_X(\underline{\theta}) \left[ (1 + \tau^{1-\sigma})^{\frac{1}{\sigma-\gamma(\sigma-1)}} \tilde{\pi}_H(\underline{\theta}) + \left( w - f_x Y^{\frac{1}{\sigma-\gamma(\sigma-1)}} \right) r \right] + (1 - \tilde{I}_X(\underline{\theta})) \tilde{\pi}_H(\underline{\theta}) \right\} \quad (17)$$

Notice that through equation (16) I am assuming that the most productive agents are exporting as otherwise we simply have no trade and we are back to the closed economy. On the other hand equation (17) leaves open the possibility of least able agents exporting, or whether the extensive margin of trade exists or not.

With this change, the proof of Lemma 1 would proceed exactly as before. Lemma 2 also holds, because the property  $tb_2(a, \theta) = b_1(ta, \theta)$  still holds. The reason essentially is that  $\tilde{w}(\theta)$  and  $w_e^0(\theta)$  move in proportion to  $Y^{\frac{1}{\sigma-\gamma(\sigma-1)}}$  and  $\theta_{ue}$  is independent of  $Y$ . Defining the  $\zeta$  function now as

$$\zeta_o(\lambda, Y) = \left[ L \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{w}}^{\bar{w}} (\theta k_v(w, \theta; \lambda, Y))^{\frac{\gamma(\sigma-1)}{\sigma}} \left\{ 1 + I_X(w, \theta; \lambda, Y) \left[ (1 + \tau^{1-\sigma})^{\frac{1}{\sigma}} - 1 \right] \right\} dG(w; \lambda, Y) dF(\theta) \right]^{\frac{\sigma}{\sigma-1}}$$

proofs of Lemmas 3-5 and Propositions 1 and 2 would follow the same logic as in the closed economy.

Before proving Proposition 3 I establish some intermediate lemmas.

**Lemma 6.** *For any fixed  $\lambda \geq 1$  and  $Y$  the invariant wealth distribution in the open economy first order stochastically dominates the distribution in the closed economy.*

*Proof.* Let  $W_c = [\underline{w}_c, \bar{w}_c]$  and  $W_o = [\underline{w}_o, \bar{w}_o]$  be the ergodic sets in the closed and open economy, respectively. Define  $W = [\underline{w}, \bar{w}]$  where  $\underline{w} = \underline{w}_c$  and  $\bar{w} = \bar{w}_o$ . Let  $P_c, P_o : W \times \mathcal{W} \rightarrow [0, 1]$  be the closed and open economy transition functions defined on the new space just as in the proof of Lemma 1. It is easy to check that these transitions functions satisfy all the properties required for the existence of a unique stationary distribution on  $(W, \mathcal{W})$ . Thus to prove the claim it is sufficient to show that  $T_o^*$  dominates  $T_c^*$ . The reasoning would be the same as in the proof of Lemma 4 and the result would follow from the fact that  $b_o(s, \theta) \geq b_c(s, \theta), \forall s \in W$ .  $\square$

**Lemma 7.** *For any  $\lambda \geq 1, c_o(\lambda) > c_c(\lambda)$ .*

*Proof.* The proof is similar to that of Lemma 5, using the fact that for all  $(w, \theta)$  revenues are (weakly) higher in the open economy and Lemma 6. The strict inequality follows because exporting agents have strictly higher revenues than they would have in the closed economy.  $\square$

**Proposition (3, reminded).** *For any fixed  $\lambda \geq 1$  the wealth distribution in the stationary equilibrium of the open economy first order stochastically dominates the distribution in the stationary equilibrium of the closed economy. Aggregate output of the differentiated sector and aggregate capital are higher in the open economy:  $Y_o^*(\lambda) > Y_c^*(\lambda), K_o(\lambda) > K_c(\lambda)$ .*

*Proof.* Combining Corollary 2 and Lemma 7 we get  $\frac{Y_o^*(\lambda)}{Y_c^*(\lambda)} = \left[ \frac{c_o(\lambda)}{c_c(\lambda)} \right]^{\frac{\sigma-\gamma(\sigma-1)}{\sigma(1-\gamma)}} > 1$ . An implication of Corollary 1 is that  $\mathcal{G}_o(\cdot; \lambda, Y_2) \succeq \mathcal{G}_o(\cdot; \lambda, Y_1)$  for  $Y_2 \geq Y_1$ . Combining this with Lemma 6 we

get  $\mathcal{G}_o(\cdot; \lambda, Y_o^*(\lambda)) \succeq \mathcal{G}_o(\cdot; \lambda, Y_c^*(\lambda)) \succeq \mathcal{G}_c(\cdot; \lambda, Y_c^*(\lambda))$ . It follows immediately that  $K_o(\lambda) = \int w \mathcal{G}_o(dw; \lambda, Y_o^*(\lambda)) \geq \int w \mathcal{G}_c(dw; \lambda, Y_c^*(\lambda)) = K_c(\lambda)$ . In fact since  $Y_o^*(\lambda) > Y_c^*(\lambda)$  we have a strict inequality in the previous expression.  $\square$

Now I establish a lemma useful for proving Proposition 4.

**Lemma 8.** *Suppose  $f_x = 0$ . Fix  $Y$  and  $\lambda$ . Let  $\mathcal{G}_o$  and  $\mathcal{G}_c$  be the invariant measures corresponding to the open and closed economy, respectively. Let  $\eta(w) = \frac{1}{u}w$  where  $u = (1 + \tau^{1-\sigma})^{\frac{1}{\sigma-\gamma(\sigma-1)}} > 1$ . Then  $\mathcal{G}_c(A) = \mathcal{G}_o(\eta^{-1}(A))$  for all  $A \in \mathcal{W}_c$ .*

*Proof.* With  $f_x = 0$  all entrepreneurs export. Denoting by  $W_c = [w_c, \bar{w}_c]$  and  $W_o = [w_o, \bar{w}_o]$  the ergodic sets in the closed and open economy we have  $w_o = uw_c$  and  $\bar{w}_o = u\bar{w}_c$ . Then it might be verified that for all  $w \in W_o$  we have  $b_o(w, \theta) = ub_c(\frac{w}{u}, \bar{\theta})$ . The rest of the proof then uses the fixed point property of the invariant measures and change of variables theorem, similarly as in the proof of Lemma 2.  $\square$

**Corollary 3.** *Suppose  $f_x = 0$ . Fix  $Y$  and  $\lambda$ . Let  $G_o$  and  $G_c$  be distribution functions for the invariant measures corresponding to the open and closed economy, respectively. Then  $G_c(w) = G_o(uw)$ , where  $u = (1 + \tau^{1-\sigma})^{\frac{1}{\sigma-\gamma(\sigma-1)}} > 1$ .*

**Lemma 9.** *Suppose  $f_x = 0$ . Then  $\frac{c_o(\lambda)}{c_c(\lambda)} = u^{\frac{\sigma}{\sigma-1}}$  where  $u = (1 + \tau^{1-\sigma})^{\frac{1}{\sigma-\gamma(\sigma-1)}} > 1$ .*

*Proof.* Follows from direct computation using expressions for  $k_v$ , Corollary 3 and simple change of variables.  $\square$

**Proposition** (4, reminded). *Suppose there are no fixed costs of exporting ( $f_x = 0$ ). Then for any  $\lambda \geq 1$  the wealth distribution in the open economy is just a scaled-up version of the distribution in the closed economy, where the scaling does not depend on  $\lambda$ . More precisely,  $G_c(w; \lambda, Y_c^*(\lambda)) = G_o\left(\left(1 + \tau^{1-\sigma}\right)^{\frac{1}{(\sigma-1)(1-\gamma)}} w; \lambda, Y_o^*(\lambda)\right)$ .*

*Proof.* From Corollary 1 and equation (13) we can write  $G_o(w; Y_o^*(\lambda)) = G_o\left(c_o(\lambda)^{-\frac{1}{\sigma(1-\gamma)}} w; 1\right)$  and similarly  $G_c(w; Y_c^*(\lambda)) = G_c\left(c_c(\lambda)^{-\frac{1}{\sigma(1-\gamma)}} w; 1\right)$ . Using Corollary 3 we obtain

$$G_c(w; Y_c^*(\lambda)) = G_c\left(c_c(\lambda)^{-\frac{1}{\sigma(1-\gamma)}} w; 1\right) = G_o\left(u c_c(\lambda)^{-\frac{1}{\sigma(1-\gamma)}} w; 1\right) = G_o\left(u \left(\frac{c_o(\lambda)}{c_c(\lambda)}\right)^{\frac{1}{\sigma(1-\gamma)}} w, Y_o^*(\lambda)\right)$$

Finally, using Lemma 9 we get:

$$G_c(w; Y_c^*(\lambda)) = G_o\left(u u^{\frac{\sigma}{\sigma-1} \frac{1}{\sigma(1-\gamma)}} w; Y_o^*(\lambda)\right) = G_o\left(\left(1 + \tau^{1-\sigma}\right)^{\frac{1}{(\sigma-1)(1-\gamma)}} w; \lambda, Y_o^*(\lambda)\right)$$

$\square$

**Corollary 4.** *Suppose there are no fixed costs of exporting ( $f_x = 0$ ). Then aggregate losses from credit frictions are the same in the open and closed economy and gains from trade are independent of the degree of frictions:  $\frac{Y_o^*(\lambda)}{Y_c^*(\lambda)} = \frac{K_o(\lambda)}{K_c(\lambda)} = (1 + \tau^{1-\sigma})^{\frac{1}{(\sigma-1)(1-\gamma)}}$ .*

*Proof.* That  $\frac{K_o(\lambda)}{K_c(\lambda)} = (1 + \tau^{1-\sigma})^{\frac{1}{(\sigma-1)(1-\gamma)}}$  follows immediately from 4. To compare equilibrium outputs evaluate:

$$\frac{Y_o^*(\lambda)}{Y_c^*(\lambda)} = \left[ \frac{c_o(\lambda)}{c_c(\lambda)} \right]^{\frac{\sigma-\gamma(\sigma-1)}{\sigma(1-\gamma)}} = u^{\frac{\sigma}{\sigma-1} \frac{\sigma-\gamma(\sigma-1)}{\sigma(1-\gamma)}} = u^{\frac{\sigma-\gamma(\sigma-1)}{(\sigma-1)(1-\gamma)}} = \left(1 + \tau^{1-\sigma}\right)^{\frac{1}{(\sigma-1)(1-\gamma)}}$$

□

## A.3 Extensions

### A.3.1 Correlated Ability Shocks

Results shown with the i.i.d. distributed ability could be extended to the case with correlated shocks. Specifically, replacing Assumption 2 with Assumption 4 would give results corresponding to the those from the previous sections, with the difference that we would have to keep track of the joint wealth-ability distribution.

**Assumption 4.** *Entrepreneurial ability shocks  $\theta$  within a lineage follow an exogenous Markov process summarized by the measurable space  $(Z, \mathcal{Z})$ , where  $Z = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$  and  $\mathcal{Z}$  is the Borel  $\sigma$ -algebra on  $Z$ , and the transition function  $Q$ .  $Q$  is increasing, and satisfies the following property: for some  $\omega > 0$*

$$Q(\theta, (a, b]) \geq \omega(b - a), \quad \forall \theta \in Z, \forall \underline{\theta} \leq a \leq b \leq \bar{\theta} \quad (18)$$

The assumption of  $Q$  increasing means that probability of receiving high ability draw tomorrow is higher when the current draw is high. This is a formalization of persistence in the ability process. The property (18) means that that for any ability  $\theta$  the probability of eventually reaching any nondegenerate interval in  $Z$  is positive. This condition is used to generate enough mobility in the economy and it could be replaced with other conditions that achieve the same technical purpose. Assumption 4 guarantees that that there is a unique stationary measure  $\mu^*$  on  $(Z, \mathcal{Z})$ .

### A.3.2 Welfare Comparisons

I describe here in more detail how the welfare losses plotted in Figure 8 were calculated.<sup>30</sup> I concentrate on the second thought experiment, the method is analogous for the first experiment. Permanent consumption equivalent of utility of an entrepreneur with wealth  $w$  and ability  $\theta$  living in a stationary equilibrium of economy with frictions  $\lambda_1$  is defined as:

$$\frac{1}{1-\beta} \ln c_{eq}(w, \theta; \lambda_1) = v(w, \theta; \lambda_1)$$

Suppose an entrepreneur could move to the economy with  $\lambda_2 > \lambda_1$  keeping his wealth and ability, but he would have to pay a fraction of his new permanent consumption equivalent. The maximum fraction  $X(w, \theta)$  he would be willing to pay is found from:

$$v(w, \theta; \lambda_1) = \frac{1}{1-\beta} \ln [(1 - X(w, \theta)) c_{eq}(w, \theta; \lambda_2)] = \frac{1}{1-\beta} \ln (1 - X(w, \theta)) + v(w, \theta; \lambda_2)$$

<sup>30</sup>This calculation draws on Buera and Shin (2010).

As a measure of aggregate welfare losses I compute a single fraction  $X$  such that when all agents move and give up a share  $X$  of their new permanent consumption equivalent, aggregate welfare is the same as in the original equilibrium when the utilitarian welfare function is used. Denoting by  $H(w, \theta; \lambda_1)$  the stationary distribution in the original equilibrium, we have:

$$X = 1 - \exp \left\{ (1 - \beta) \left( \int [v(w, \theta; \lambda_1) - v(w, \theta; \lambda_2)] H(dw, d\theta; \lambda) \right) \right\}$$

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